Waveform Design for One-Bit Radar Systems Under Uncertain Interference Statistics

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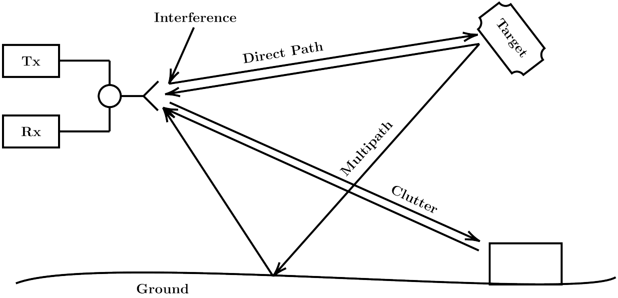
Cognitive active sensing

Clutter

refers to unwanted echoes that are usually correlated with the transmitted signal.

Interference

is the noise as well as jamming signals.



Jointly exploit Tx-Rx DoF

A natural criterion for designing transmit signals and receive filters is to maximize the signal-to-clutter-plus-interference ratio (SCIR) of the receiver output at the time of target detection.

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A natural criterion for designing transmit signals and receive filters is to maximize the signal-to-clutter-plus-interference ratio (SCIR) of the receiver output at the time of target detection.

Why?

"Pulse compression radar systems make use of transmit code sequences and receive filters that are specially designed to achieve good range resolution and target detection capability at practically acceptable transmit peak power levels." [1]

[1] P. Stoica et al., Transmit codes and receive filters for radar, 2008

How do you deal with quantization in receiver

Quantization on low bitrate?

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Quantization on low bitrate?

One-bit quantizer

- Low cost
- Low power
- Faster than traditional scalar quantizers

} Reduction in the complexity of hardware implementation

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One-bit quantizer

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} Reduction in the complexity of hardware implementation

But we lose some information

The knowledge of interference statistics are available in only a normalized sense.

The question we ask:

Q:

Given that the signal covariance matrix can only be measured in a *normalized sense*, or in other words, under uncertainties in interference statistics, can we still design the transmit signal and the receive filter coefficients, in a joint manner?

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Q: Given that the signal covariance matrix can only be measured in a *normalized sense*, or in other words, under uncertainties in interference statistics, can we still design the transmit signal and the receive filter coefficients, in a joint manner?

A: Yes, we can!

Preliminaries

Data model

$$\boldsymbol{s} = \begin{bmatrix} s_1, s_2, \cdots, s_{N-1}, s_N \end{bmatrix}^T$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{N-1} \\ y_N \end{bmatrix} = \alpha_0 \begin{bmatrix} s_1 \\ \vdots \\ s_{N-1} \\ s_N \end{bmatrix} + \alpha_1 \begin{bmatrix} 0 \\ s_1 \\ \vdots \\ s_{N-1} \end{bmatrix} + \cdots + \alpha_{N-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_1 \end{bmatrix} + \alpha_{-1} \begin{bmatrix} s_2 \\ \vdots \\ s_N \\ 0 \end{bmatrix} + \cdots + \alpha_{-N+1} \begin{bmatrix} s_N \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_{N-1} \\ \epsilon_N \end{bmatrix}$$

Preliminaries (contd.)

Data model

$$\boldsymbol{y} = \boldsymbol{A}^H \boldsymbol{\alpha} + \boldsymbol{\epsilon},$$

$$\boldsymbol{A}^H = \begin{bmatrix} s_1 & 0 & \cdots & 0 & s_N & s_{N-1} & \cdots & s_2 \\ s_2 & s_1 & & \vdots & 0 & s_N & & \vdots \\ \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \ddots & s_N \\ s_N & s_{N-1} & \cdots & s_1 & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\boldsymbol{\alpha} = [\alpha_0,\ \alpha_1,\ \cdots,\ \alpha_{N-1},\ \alpha_{-N+1},\ \cdots,\ \alpha_{-1}]^T \in \mathbb{C}^{2N-1}.$$

Preliminaries (contd.)

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$$\boldsymbol{\alpha} = [\alpha_0,\ \alpha_1,\ \cdots,\ \alpha_{N-1},\ \alpha_{-N+1},\ \cdots,\ \alpha_{-1}]^T \in \mathbb{C}^{2N-1}.$$

Keep in mind

$$\beta \triangleq \mathbb{E}\{|\alpha_k|^2\},\ k \neq 0, \qquad \boldsymbol{\Gamma} \triangleq \mathbb{E}\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^H\}.$$

Filters

We are after the principal RCS: α_0

- MF estimate: $\hat{\alpha}_0 = \dfrac{\boldsymbol{s}^H \boldsymbol{y}}{\|\boldsymbol{s}\|^2}$

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$$\mathrm{MSE}(\hat{\alpha}_0) = \mathbb{E}\left\{ \left| \frac{\boldsymbol{w}^H \boldsymbol{y}}{\boldsymbol{w}^H \boldsymbol{s}} - \alpha_0 \right|^2 \right\}.$$

The criterion

Mean square error

$$\mathrm{MSE}(\hat{\alpha}_0) = \mathbb{E}\left\{\left|\frac{\boldsymbol{w}^H\boldsymbol{y}}{\boldsymbol{w}^H\boldsymbol{s}} - \alpha_0\right|^2\right\} = \frac{\boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w}}{|\boldsymbol{w}^H\boldsymbol{s}|^2}$$

$$\boldsymbol{R} = \beta \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \boldsymbol{J}_k \boldsymbol{s}\boldsymbol{s}^H \boldsymbol{J}_k^H + \boldsymbol{\Gamma},$$

$$\mathbf{J}_k = \mathbf{J}_{-k}^H = \begin{bmatrix} 0 & \dots & 0 & 1 & \dots & 0 \\ \vdots & & & & \ddots & \\ & & & & & 1 \\ \underbrace{0 \quad \dots \quad 0}_{k} & & & \dots & & \end{bmatrix}_{N\times N}^H, k = 0, 1, \cdots, N-1.$$

Primary goal

The optimization criteria

$$\min_{\boldsymbol{w},\boldsymbol{s}} \frac{\boldsymbol{w}^H \boldsymbol{R} \boldsymbol{w}}{|\boldsymbol{w}^H \boldsymbol{s}|^2}$$

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Cognitive REceiver and Waveform design (CREW)

- `CREW (gra)`[1]
- `CREW (fre)`[1]
- `CREW (mat)`[1]
- `CREW (cyclic)`[2]

[1] P. Stoica et al. Optimization of the Receive Filter and Transmit Sequence for Active Sensing, 2012.

[2] M. Soltanalian et al. Joint Design of the Receive Filter and Transmit Sequence for Active Sensing, 2013.

CREW (cyclic)

For fixed $\boldsymbol{s}$:

$$\hat{\boldsymbol{w}} = \boldsymbol{R}^{-1}\boldsymbol{s}$$

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For fixed $\boldsymbol{w}$:

$$\begin{aligned}\boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w} &= \boldsymbol{w}^H\left(\beta\sum_{\substack{k=-N+1\\k\neq 0}}^{N-1}\boldsymbol{J}_k\boldsymbol{s}\boldsymbol{s}^H\boldsymbol{J}_k^H + \Gamma\right)\boldsymbol{w}\\ &= \boldsymbol{s}^H\underbrace{\left(\beta\sum_{\substack{k=-N+1\\k\neq 0}}^{N-1}\boldsymbol{J}_k\underbrace{\boldsymbol{w}\boldsymbol{w}^H}_{\boldsymbol{W}}\boldsymbol{J}_k^H\right)}_{\boldsymbol{Q}}\boldsymbol{s} + \underbrace{\boldsymbol{w}^H\Gamma\boldsymbol{w}}_{\mu}.\end{aligned}$$

CREW (cyclic) (contd.)

$$\mathrm{MSE}(\hat{\alpha}_0) = \frac{\boldsymbol{s}^H \boldsymbol{Q} \boldsymbol{s} + \mu}{\boldsymbol{s}^H \boldsymbol{W} \boldsymbol{s}} \triangleq \frac{a(\boldsymbol{s})}{b(\boldsymbol{s})} = f(\boldsymbol{s}) \qquad \text{[Fractional program]}$$

CREW (cyclic) (contd.)

$$\mathrm{MSE}(\hat{\alpha}_0) = \frac{\boldsymbol{s}^H \boldsymbol{Q} \boldsymbol{s} + \mu}{\boldsymbol{s}^H \boldsymbol{W} \boldsymbol{s}} \triangleq \frac{a(\boldsymbol{s})}{b(\boldsymbol{s})} = f(\boldsymbol{s}) \qquad \text{[Fractional program]}$$

Can be recast as (see [1]):

$$\max_{\boldsymbol{s}} \ \boldsymbol{s}^H \tilde{\boldsymbol{T}} \boldsymbol{s} \qquad \text{s.t. } |s_k| = 1, \qquad 1 \leq k \leq N$$

where

$$\boldsymbol{T} \triangleq \boldsymbol{Q} + (\mu/N)\boldsymbol{I} - f(\boldsymbol{s}_*)\boldsymbol{W}$$
$$\tilde{\boldsymbol{T}} \triangleq \lambda \boldsymbol{I} - \boldsymbol{T}$$

[1] M. Soltanalian et al. Joint Design of the Receive Filter and Transmit Sequence for Active Sensing, 2013.

CREW (cyclic) (contd.)

Power method-like iterations

$$
\begin{aligned}
&\min_{\boldsymbol{s}^{(t+1)}} \left\| \boldsymbol{s}^{(t+1)} - \tilde{\boldsymbol{T}} \boldsymbol{s}^{(t)} \right\|_2 \\
&\text{s.t.} \ \left| s_k^{(t+1)} \right| = 1, \qquad 1 \leq k \leq N.
\end{aligned}
$$

CREW (cyclic) (contd.)

Power method-like iterations

$$\begin{aligned} \min_{\boldsymbol{s}^{(t+1)}} \quad & \left\| \boldsymbol{s}^{(t+1)} - \tilde{\boldsymbol{T}} \boldsymbol{s}^{(t)} \right\|_2 \\ \text{s.t.} \quad & \left| s_k^{(t+1)} \right| = 1, \qquad 1 \leq k \leq N. \end{aligned}$$

The solution is simply given analytically by $\boldsymbol{s}^{(t+1)} = e^{j \arg(\tilde{\boldsymbol{T}} \boldsymbol{s}^{(t)})}$.

CREW (cyclic) (contd.)

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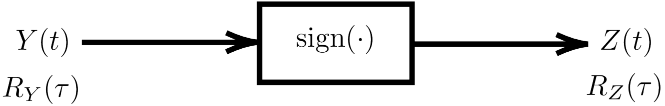
The solution is simply given analytically by $\boldsymbol{s}^{(t+1)} = e^{j \arg(\tilde{\boldsymbol{T}} \boldsymbol{s}^{(t)})}$.

Everything is fine so far.

One-bit receiver

$Y(t)$, a Real-valued, scalar, stationary Gaussian process



$$R_Z(\tau) \triangleq \mathbb{E}\{Z(t+\tau)Z(t)\} = \frac{2}{\pi}\sin^{-1}\bar{R}_Y(\tau)$$

where

$$\bar{R}_Y(\tau) \triangleq R_Y(\tau)/R_Y(0).$$

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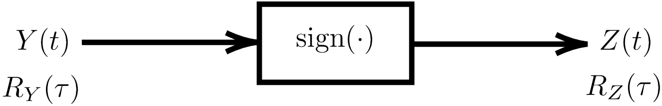

$$R_Z(\tau) \triangleq \mathbb{E}\{Z(t+\tau)Z(t)\} = \frac{2}{\pi}\sin^{-1}\bar{R}_Y(\tau)$$

where

$$\bar{R}_Y(\tau) \triangleq R_Y(\tau)/R_Y(0).$$

Bussgang theorem

$R_{ZY}(\tau) = \zeta R_Y(\tau)$ where ζ depends on the power of the process $Y(t)$.

One-bit receiver (contd.)

Complex-valued vector process γ

$$\gamma = \frac{1}{\sqrt{2}}\mathrm{csign}(\boldsymbol{y}) \triangleq \frac{1}{\sqrt{2}}[\mathrm{sign}(\mathrm{Re}(\boldsymbol{y})) + j\mathrm{sign}(\mathrm{Im}(\boldsymbol{y}))].$$

Following still holds:

$$\bar{\boldsymbol{R}}_{\boldsymbol{y}} = \sin\left(\frac{\pi}{2}\boldsymbol{R}_{\gamma}\right),$$

where the normalized auto-correlation matrix of $\boldsymbol{y}$ is given as

$$\bar{\boldsymbol{R}}_{\boldsymbol{y}} \triangleq \boldsymbol{W}^{-\frac{1}{2}}\boldsymbol{R}_{\boldsymbol{y}}\boldsymbol{W}^{-\frac{1}{2}},$$

and where $\boldsymbol{W} = \boldsymbol{R}_{\boldsymbol{y}} \odot \boldsymbol{I}$.

Problem formulation

$\boldsymbol{R}$ is measured as $\bar{\boldsymbol{R}} = \boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{R}\boldsymbol{D}^{-\frac{1}{2}}$, where $\boldsymbol{D} = \boldsymbol{R} \odot \boldsymbol{I}$.

Problem formulation

$$\boxed{\boldsymbol{R} \text{ is measured as } \bar{\boldsymbol{R}} = \boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{R}\boldsymbol{D}^{-\frac{1}{2}}, \text{ where } \boldsymbol{D} = \boldsymbol{R} \odot \boldsymbol{I}.}$$

A meaningful approach to consider:

$$\min_{\boldsymbol{w},\boldsymbol{s}} \quad \mathbb{E}\left\{\frac{\boldsymbol{w}^H\boldsymbol{D}^{\frac{1}{2}}\bar{\boldsymbol{R}}\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{w}}{|\boldsymbol{w}^H\boldsymbol{s}|^2}\right\},$$

in which the expectation is taken over $\boldsymbol{d} = \mathrm{diag}(\boldsymbol{D})$.

Problem formulation (contd.)

We follow similar framework as CREW (cyclic).[1]
For fixed $\boldsymbol{w}$,

$$\boldsymbol{w}^H \boldsymbol{R} \boldsymbol{w} = \boldsymbol{w}^H \left(\beta \sum_{\substack{k=-N+1 \\ k \neq 0}}^{N-1} \boldsymbol{J}_k \boldsymbol{s} \boldsymbol{s}^H \boldsymbol{J}_k^H + \boldsymbol{\Gamma} \right) \boldsymbol{w}$$

$$= \boldsymbol{s}^H \left(\underbrace{\beta \sum_{\substack{k=-N+1 \\ k \neq 0}}^{N-1} \boldsymbol{J}_k \underbrace{\boldsymbol{w} \boldsymbol{w}^H}_{\boldsymbol{W}} \boldsymbol{J}_k^H}_{\chi} \right) \boldsymbol{s} + \boldsymbol{w}^H \boldsymbol{\Gamma} \boldsymbol{w}.$$

[1] M. Soltanalian et al. Joint Design of the Receive Filter and Transmit Sequence for Active Sensing, 2013.

Problem formulation (contd.)

The MSE formulation becomes,

$$\frac{\mathrm{MSE}(\hat{\alpha}_0)}{\beta} = \frac{\boldsymbol{s}^H \boldsymbol{\chi} \boldsymbol{s} + \mu}{\boldsymbol{s}^H \boldsymbol{W} \boldsymbol{s}} = f(\boldsymbol{s}), \text{ where } \mu = (\boldsymbol{w}^H \boldsymbol{\Gamma} \boldsymbol{w})/\beta.$$

Problem formulation (contd.)

The MSE formulation becomes,

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The same power method-like iterations still hold:

$$\begin{aligned} \min_{\boldsymbol{s}^{(t+1)}} & \left\| \boldsymbol{s}^{(t+1)} - \tilde{\boldsymbol{T}} \boldsymbol{s}^{(t)} \right\|_2 \\ \text{s.t.} & \left| s_k^{(t+1)} \right| = 1, \qquad 1 \leq k \leq N. \end{aligned}$$

where

$$\boldsymbol{T} \triangleq \chi - f(\boldsymbol{s}_*) \boldsymbol{W}$$
$$\tilde{\boldsymbol{T}} \triangleq \lambda \boldsymbol{I} - \boldsymbol{T}$$

Problem formulation (contd.)

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where

$$\begin{aligned} \boldsymbol{T} &\triangleq \boldsymbol{\chi} - f(\boldsymbol{s}_*) \boldsymbol{W} \\ \tilde{\boldsymbol{T}} &\triangleq \lambda \boldsymbol{I} - \boldsymbol{T} \end{aligned}$$

The solution is simply given analytically by $\boldsymbol{s}^{(t+1)} = e^{j \arg(\tilde{\boldsymbol{T}} \boldsymbol{s}^{(t)})}$.

Problem formulation (contd.)

For fixed $\boldsymbol{s}$:

$$
\begin{aligned}
\mathbb{E}\left\{\frac{\boldsymbol{w}^H \boldsymbol{D}^{\frac{1}{2}} \bar{\boldsymbol{R}} \boldsymbol{D}^{\frac{1}{2}} \boldsymbol{w}}{|\boldsymbol{w}^H \boldsymbol{s}|^2}\right\} &= \frac{\mathbb{E}\left\{\mathrm{tr}\left(\boldsymbol{w}\boldsymbol{w}^H \boldsymbol{D}^{\frac{1}{2}} \bar{\boldsymbol{R}} \boldsymbol{D}^{\frac{1}{2}}\right)\right\}}{|\boldsymbol{w}^H \boldsymbol{s}|^2} \\
&= \frac{\mathbb{E}\left\{\boldsymbol{d}^H \left(\boldsymbol{w}\boldsymbol{w}^H \odot \bar{\boldsymbol{R}}^H\right) \boldsymbol{d}\right\}}{|\boldsymbol{w}^H \boldsymbol{s}|^2} \\
&= \frac{\mathrm{tr}\left(\left(\boldsymbol{w}\boldsymbol{w}^H \odot \bar{\boldsymbol{R}}^H\right) \mathbb{E}\left\{\boldsymbol{d}\boldsymbol{d}^H\right\}\right)}{|\boldsymbol{w}^H \boldsymbol{s}|^2}.
\end{aligned}
$$

$$
\boxed{\boldsymbol{d} \rightarrowtail \boldsymbol{D} \rightarrowtail \boldsymbol{R} \rightarrowtail \{\beta, \Gamma\}}
$$

Problem formulation (contd.)

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$$\bar{\Gamma} = \boldsymbol{A}^{-\frac{1}{2}} \Gamma \boldsymbol{A}^{-\frac{1}{2}},$$

Then $\boldsymbol{R}$ becomes,

$$\boldsymbol{R} = \boldsymbol{D}^{\frac{1}{2}} \bar{\boldsymbol{R}} \boldsymbol{D}^{\frac{1}{2}} = \beta \boldsymbol{S} + \boldsymbol{A}^{\frac{1}{2}} \bar{\Gamma} \boldsymbol{A}^{\frac{1}{2}}. \qquad [\boldsymbol{S} = \sum_{k \neq 0} \boldsymbol{J}_k \boldsymbol{s}\boldsymbol{s}^H \boldsymbol{J}_k^H]$$

Problem formulation (contd.)

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Solve,

$$\{\hat{\boldsymbol{d}}, \hat{\boldsymbol{a}}, \hat{\beta}\} = \arg \min_{\boldsymbol{d}, \boldsymbol{a}, \beta} \Big\| \mathrm{Diag}(\boldsymbol{d})^{\frac{1}{2}} \ \bar{\boldsymbol{R}} \ \mathrm{Diag}(\boldsymbol{d})^{\frac{1}{2}} - \beta \boldsymbol{S} - \mathrm{Diag}(\boldsymbol{a})^{\frac{1}{2}} \ \bar{\Gamma} \ \mathrm{Diag}(\boldsymbol{a})^{\frac{1}{2}} \Big\|_F^2, \quad \text{s.t.} \quad \boldsymbol{d} > \mathbf{0}, \boldsymbol{a} > \mathbf{0}, \beta > 0. \tag{1}$$

Problem formulation (contd.)

Γ can be measured in a similar way in a *normalized sense*.

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Solve,

$$\{\hat{\boldsymbol{d}}, \hat{\boldsymbol{a}}, \hat{\beta}\} = \arg \min_{\boldsymbol{d}, \boldsymbol{a}, \beta} \Big\| \mathrm{Diag}(\boldsymbol{d})^{\frac{1}{2}} \ \bar{\boldsymbol{R}} \ \mathrm{Diag}(\boldsymbol{d})^{\frac{1}{2}} - \beta \boldsymbol{S} - \mathrm{Diag}(\boldsymbol{a})^{\frac{1}{2}} \ \bar{\Gamma} \ \mathrm{Diag}(\boldsymbol{a})^{\frac{1}{2}} \Big\|_F^2, \quad \text{s.t.} \ \ \boldsymbol{d} > \boldsymbol{0}, \boldsymbol{a} > \boldsymbol{0}, \beta > 0. \tag{1}$$

- Non-convex and hard problem to solve,
- Can be solved in an alternative manner,
- Solution will provide β and $\boldsymbol{d}$ in an average sense.

Problem formulation (contd.)

Lets EVD: $\boldsymbol{d}\boldsymbol{d}^H = \sum_{k=1}^{N} \nu_k \boldsymbol{u}_k \boldsymbol{u}_k^H$

Problem formulation (contd.)

Lets EVD: $\boldsymbol{d}\boldsymbol{d}^H = \sum_{k=1}^{N} \nu_k \boldsymbol{u}_k \boldsymbol{u}_k^H$

$$\begin{aligned}
&\operatorname{tr}\left(\left(\boldsymbol{w}\boldsymbol{w}^H \odot \bar{\boldsymbol{R}}^H\right) \sum_{k=1}^{N} \nu_k \boldsymbol{u}_k \boldsymbol{u}_k^H\right) \\
&= \sum_{k=1}^{N} \nu_k \boldsymbol{u}_k^H \left(\boldsymbol{w}\boldsymbol{w}^H \odot \bar{\boldsymbol{R}}^H\right) \boldsymbol{u}_k \\
&= \operatorname{tr}\left(\left(\boldsymbol{w}\boldsymbol{w}^H\right) \sum_{k=1}^{N} \nu_k \operatorname{diag}(\boldsymbol{u}_k)\, \bar{\boldsymbol{R}} \operatorname{diag}\left(\boldsymbol{u}_k^H\right)\right) \\
&= \boldsymbol{w}^H \boldsymbol{Q} \boldsymbol{w},
\end{aligned}$$

where

$$\boldsymbol{Q} = \sum_{k=1}^{N} \nu_k \operatorname{diag}(\boldsymbol{u}_k)\, \bar{\boldsymbol{R}} \operatorname{diag}(\boldsymbol{u}_k^H).$$

Problem formulation (contd.)

Finally, for fixed $\boldsymbol{s}$:

$$\min_{\boldsymbol{w},\boldsymbol{s}} \quad \frac{\boldsymbol{w}^H \boldsymbol{Q} \boldsymbol{w}}{|\boldsymbol{w}^H \boldsymbol{s}|^2}.$$

which has a closed-from solution:

$$\hat{\boldsymbol{w}} = \boldsymbol{Q}^{-1} \boldsymbol{s},$$

within a multiplicative constant.

CREW (one-bit)

Step 0: Initialize $\boldsymbol{s}$ as a unimodular (or low PAR) vector in $\mathbb{C}^N$, $\boldsymbol{w}$ as a random vector in $\mathbb{C}^N$, and the outer loop index $t = 1$.

Step 1: For fixed $\boldsymbol{w}$,

Step 1.1: Compute χ, $\boldsymbol{W}$, and in turn $\tilde{\boldsymbol{T}}$,

Step 1.2: Solve the power method like iterations and calculate $\boldsymbol{s}^{(t)}$ in each iteration until convergence.

Step 2: Measure $\bar{\Gamma}$ and compute $\bar{\boldsymbol{R}}$ from the output.

Step 3: For fixed $\boldsymbol{s}$,

Step 3.1: Solve (1) to optimize $\boldsymbol{d}$,

Step 3.2: Compute the EVD of $\boldsymbol{d}\boldsymbol{d}^H$, and in turn $\boldsymbol{Q}$,

Step 3.3: Compute optimize $\boldsymbol{w}^{(t)}$ as $\boldsymbol{Q}^{-1}\boldsymbol{s}^{(t)}$.

Step 4: Repeat steps 1 to 3 until a pre-defined stop criterion is satisfied, e.g. $\left|\mathrm{MSE}^{(t+1)} - \mathrm{MSE}^{(t)}\right| < \epsilon$.

Numerical example

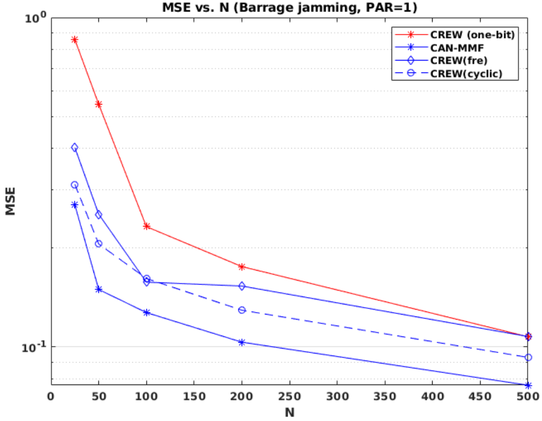

Figure: MSE vs N for different algorithms

Summary

- **We design** the **transmit signal and** the **receive filter** coefficients, in a joint manner in a scenario where the interference information can only be measured in a normalized sense, or in other words, **under uncertain**ties in **interference statistics**.
- The **knowledge of** the one-bit **measurements impacts the design of the filter** coefficients **and** in turn the **design of the filter impacts the design of transmit signal** and so on.

Thank you
and
Questions?