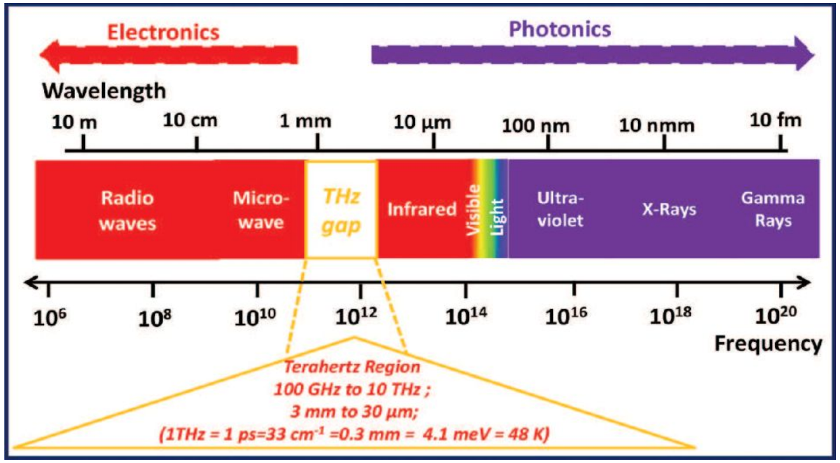
THz Multi-Layer Imaging via Nonlinear Inverse Scattering

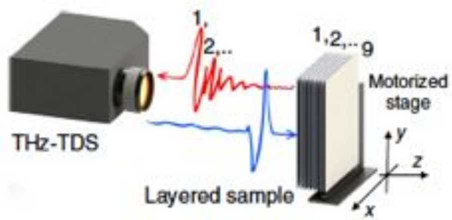
A. Bose, A. Kadu, H. Mansour, P. Wang, P. Boufounos, P. V. Orlik
Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA, USA

M. Soltanalian
University of Illinois at Chicago, Chicago, USA



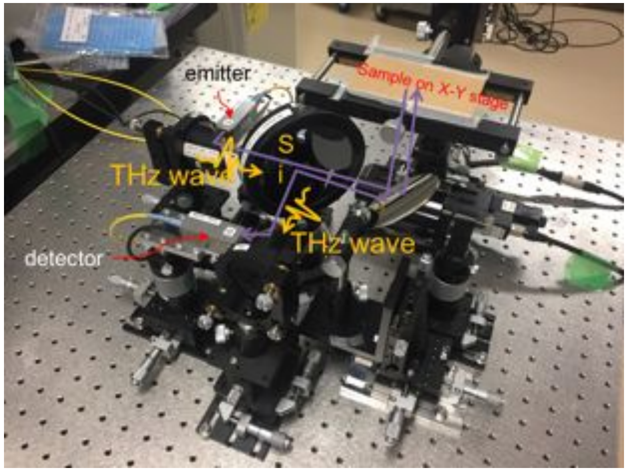
THz-TDS (THz Time-Domain Spectroscopy)

❑ Raster Scanning Mode

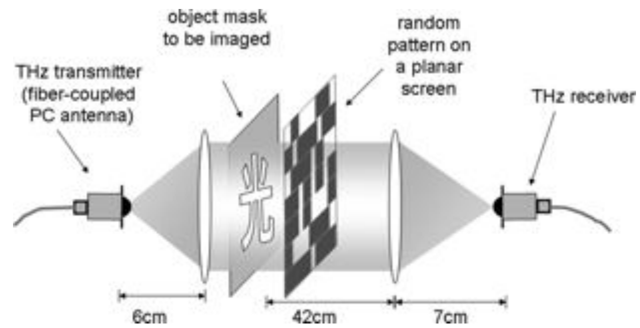


- Normal incident THz beam
- Mechanically move THz transceiver or samples
- Spot size: 1-mm radius at 1 THz
- Pulse width: 1-5 ps
- Sampling frequency: up to 5 THz
- Scanning rate: 100 Hz to 1000 Hz

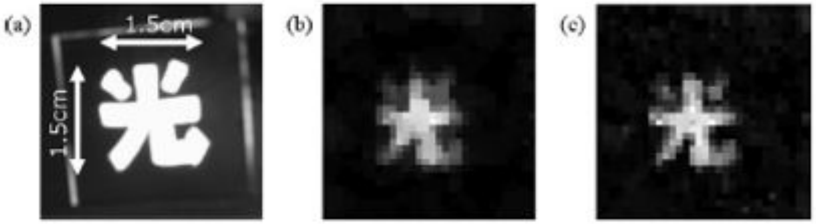
THz-TDS @ Osaka Univ.



❑ Compressed Scanning Mode

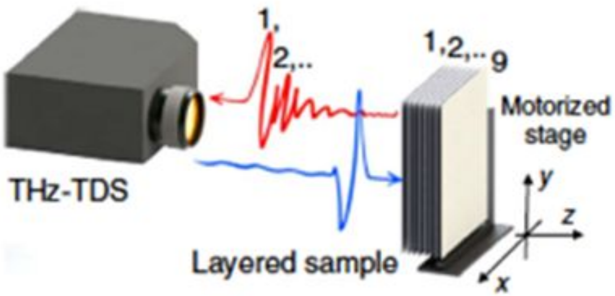


- Collimated THz beam
- Spatial light modulator (SLM) at THz band
- Focusing lens before THz detector
- No mechanical scanning
- Need random masks



THz Multi-Layer Imaging

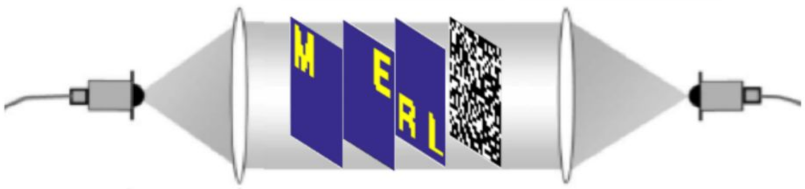
Raster Scanning

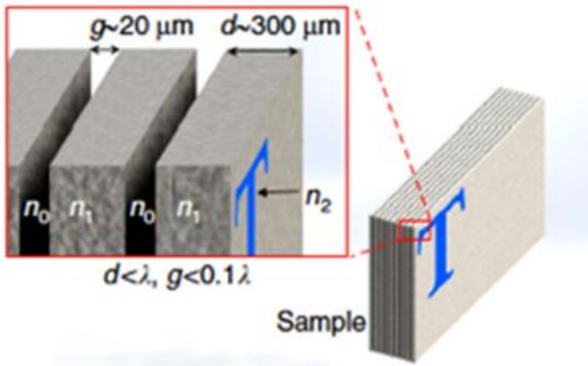


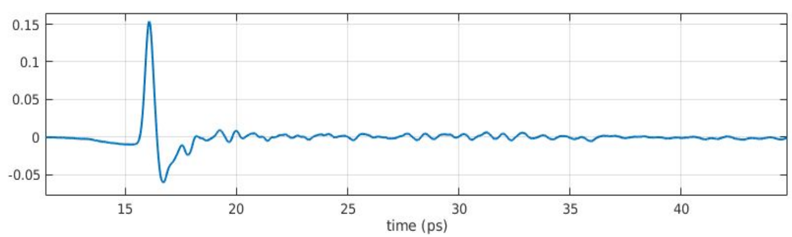
Examples:

- Books
- Old paintings
- Biomedical diagnosis
- Package inspection
- Security screening

Compressed Scanning

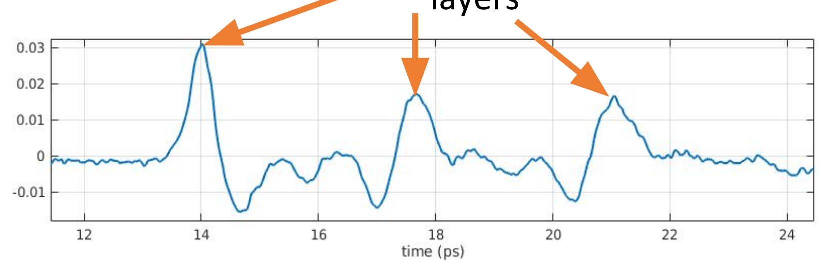






Incident signal

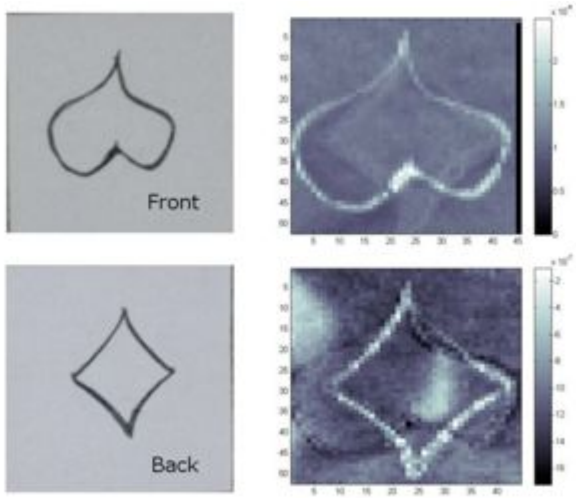
Reflections from front side of different layers



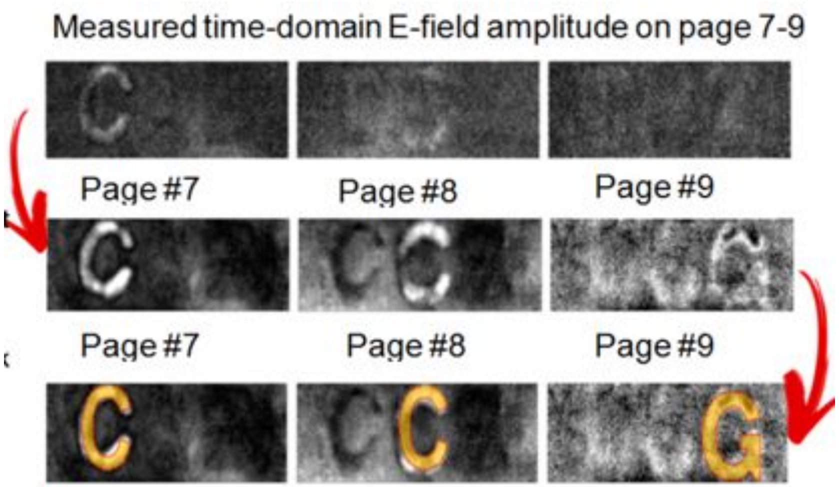
Reflected signal

Shadow Effect (I)

Ref. [1]: THz images of the front and back of the first sheet of paper



Ref. [3]: MIT THz See-Through Book



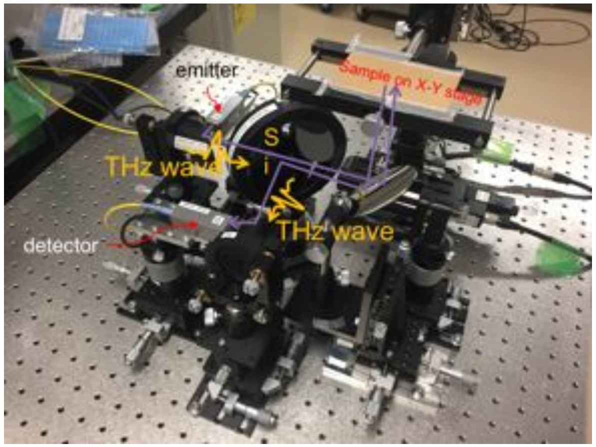
Ref. [2]: THz image (c) of the painting *La oracion en el huerto*

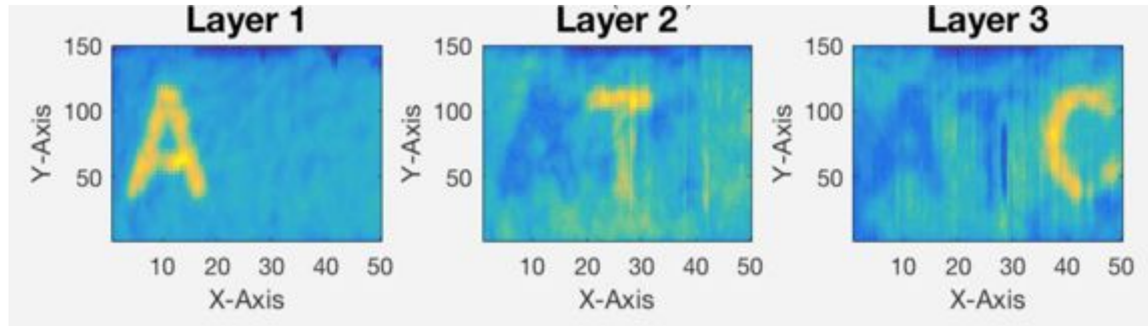
Fig. 3. a. Second detail of the painting *La oración en el huerto*. **b.** the infrared image of the detail. **c.** the THz record of the detail

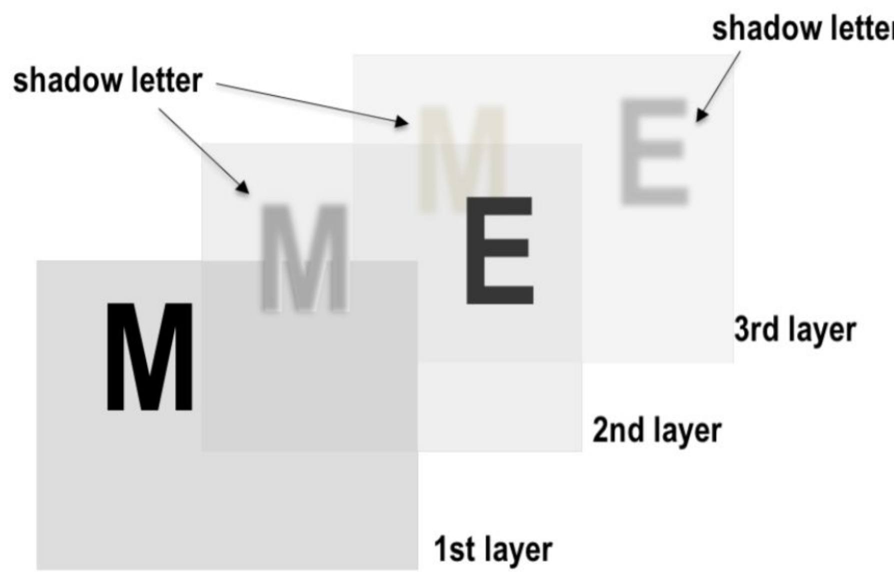
[1] G. C. Walker, et al., “Terahertz deconvolution,” Optics Express, vol. 20, no. 25, pp. 27230–27241, Dec. 2012.
[2] C. L. K. Dandolo et al., "Contribution of terahertz time-domain analysis to art history: The case of the paintings of the Santo Entierro de Nuestro Señor Jesucristo altarpiece," 42nd IRMMW-THz, Cancun, Mexico, 2017
[3] A. Redo-Sanchez, et al., “Terahertz time-gated spectral imaging for content extraction through layered structures,” Nature Communications, vol. 7, pp. 1–7, Sept. 2016.

Shadow Effect (II)

Our own experiment at Osaka University







The shadow effect of non-overlapping content for front layers to deep layers

MITSUBISHI ELECTRIC
Changes for the Better

Approach: Inverse Scattering

for a greener tomorrow

- *What is inverse scattering*: Reconstruction of the spatial permittivity of an object by probing it using electromagnetic or acoustic waves and measuring the scattered wavefield around the object.

- *Acquisition modes:*
  - Transmission mode
  - Reflection mode ←

- *How to solve inversion scattering for reflection regime:*
  - Ray-tracing model [1]
  - First order Born approximation [2]
  - Rytov approximation [3]
  - Reverse-time migration [4]

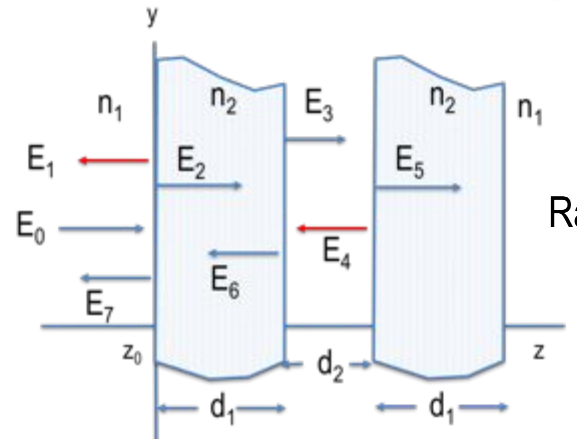
- *Problem?*
  - Fails to account for the complex interaction between the wavefield and the material properties that result in multiple scattering.
  - Requires an accurate initial target model to enable the inversion and generally suffers from poor reconstruction quality especially when the material is inhomogeneous or contains highly scattering objects.

[1] G. H. Spencer et al., “General ray-tracing procedure,” JOSA, vol. 52, no. 6, pp. 672–678, 1962.
[2] A. J. Devaney, "Inversion formula for inverse scattering within the Born approximation," Opt. Lett. 7, 111-112, 1982
[3] A. J. Devaney, “Inverse-scattering theory within the Rytov approximation,” Optics letters, vol. 6, no. 8, pp. 374–376, 1981.
[4] E. Baysal et al., “Reverse time migration,” Geophysics, vol. 48, no. 11, pp. 1514–1524, 1983.

Ray-tracing model

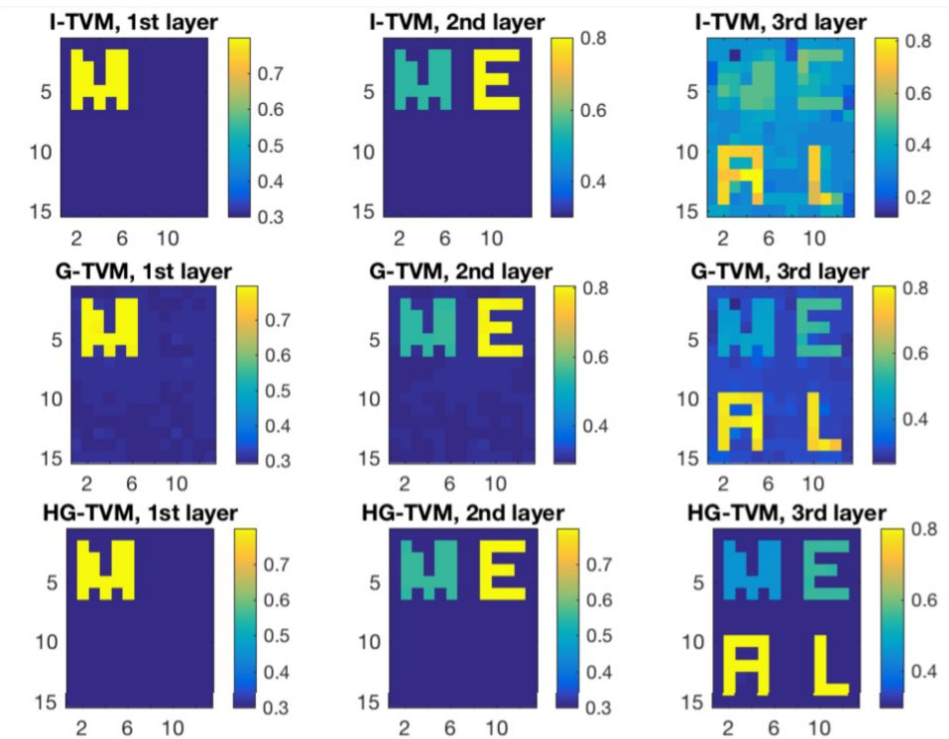
Ray-tracing model [1]

$$x(t) = \rho_1 s\left(t - 2\frac{z_0}{c}\right) + \rho_2(1-\rho_1^2)^2 s(t - 2\frac{z_0}{c} - 2\frac{n_2 d_1}{c} - 2\frac{d_2}{c})$$

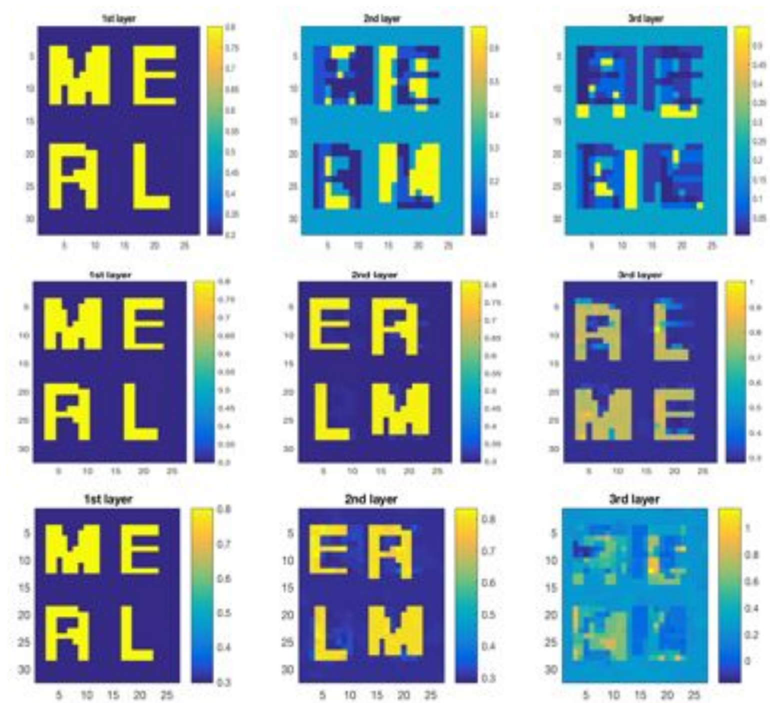


Ray Tracing [2]

Samples with non-overlapping contents



overlapping content



[1] G. H. Spencer et al., "General Ray-Tracing Procedure†," J. Opt. Soc. Am. 52, 672-678 (1962).
[2] P. Wang et al., "Multi-Layer Terahertz Imaging of Non-Overlapping Contents,“ 2018, pp. 652-656.

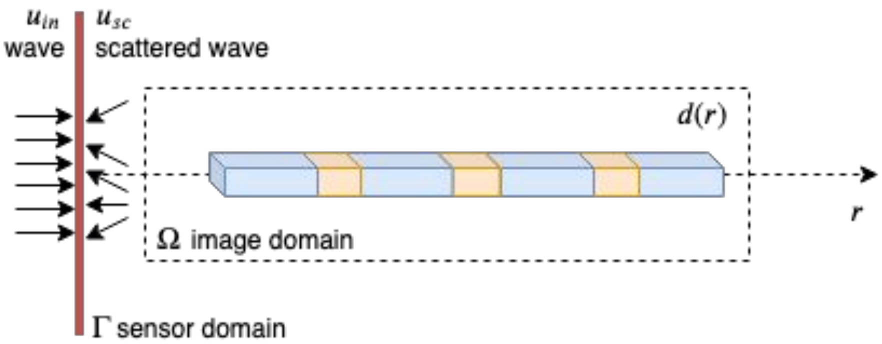
Theory of Nonlinear Inversion Model

Forward Problem:

$$u(r) = u_{in}(r) + u_{sc}(r)$$

$$u_{sc}(r) = k^2 \int_{\Omega} g(r - r')\, u(r')\, d(r')\, dr', \forall\, r \in \Omega$$

$$y(r) = \int_{\Omega} h(r - r')\, d(r') u(r')\, dr', \qquad \forall\, r \in \Gamma$$



$d(r) = \epsilon(r) - \epsilon_b$: dielectric permittivity of background

k: THz wavenumber in vacuum

$g(r) = -\frac{i}{2k_b} e^{-ik_b|r|}$: 1D free space Green's function

$k_b = k\sqrt{\epsilon_b}$: wave number of the background medium

ϵ_b: permittivity of the background

$h(r)$: Green's function of the receiver

Matrix form:

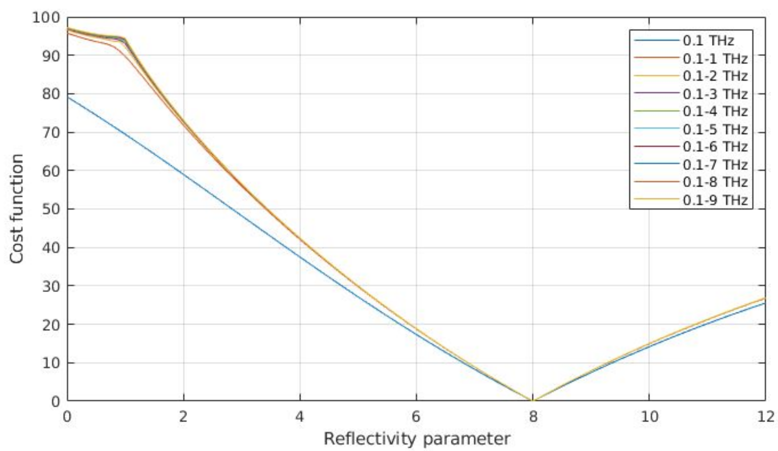
$$\boldsymbol{u}(\omega) = \boldsymbol{u}_{in}(\omega) + \boldsymbol{G}(\omega)\, \mathrm{Diag}(\boldsymbol{d})\, \boldsymbol{u}(\omega)$$

$$y(\omega) = \boldsymbol{h}^T(\omega) \mathrm{Diag}\big(\boldsymbol{u}(\omega)\big) \boldsymbol{d} + e(\omega)$$

Inverse Problem:

$$\min_{\boldsymbol{d},\boldsymbol{u}} \sum_{\omega} \frac{1}{2} \left\| y(\omega) - \boldsymbol{h}^T(\omega) \mathrm{Diag}\big(\boldsymbol{u}(\omega)\big) \boldsymbol{d} \right\|_2^2 + \mathcal{R}(\boldsymbol{d})$$

$$\text{s.t. } \boldsymbol{u}(\omega) = \big(\boldsymbol{I} - \boldsymbol{G}(\omega)\, \mathrm{Diag}(\boldsymbol{d})\big)^{-1} \boldsymbol{u}_{in}(\omega)$$



Proposed Approach

Incremental frequency inversion framework:

for $n = 1, \ldots, N_\omega$, and $\lambda_i = (0,1]$,

$$(\boldsymbol{d}_n, \boldsymbol{u}^*) \triangleq \arg\min_{d,u} \mathcal{D}_\omega(\boldsymbol{d}, \boldsymbol{u}_n) + \sum_{i=1}^{n-1} \lambda_i \mathcal{D}_i(\boldsymbol{d}, \boldsymbol{u}_i) + \mathcal{R}(\boldsymbol{d})$$

$$\text{s.t. } \boldsymbol{u}(\omega) = \left(\boldsymbol{I} - \boldsymbol{G}(\omega)\,\mathrm{Diag}(\boldsymbol{d})\right)^{-1} \boldsymbol{u}_{in}(\omega)$$

where, $\mathcal{D}_\omega(\boldsymbol{d}, \boldsymbol{u}) \triangleq \sum_\omega \frac{1}{2} \left\| y(\omega) - \boldsymbol{h}^T(\omega)\mathrm{Diag}\left(\boldsymbol{u}(\omega)\right)\boldsymbol{d} \right\|_2^2$

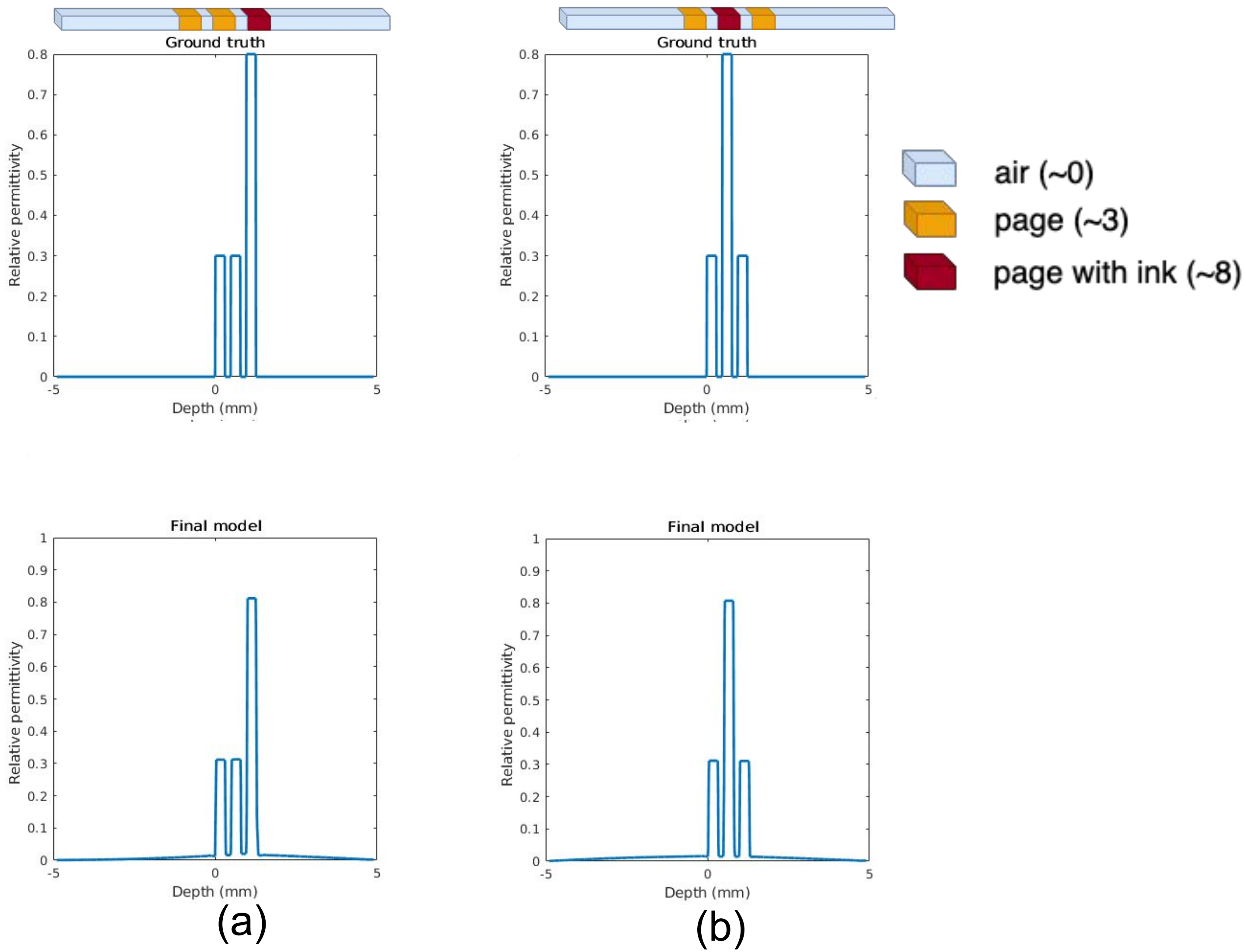
Total-variation regularization:

$\mathcal{R}(\boldsymbol{d}) \triangleq \tau \|D\boldsymbol{d}\|_1$

D: discrete finite difference operator in 1D

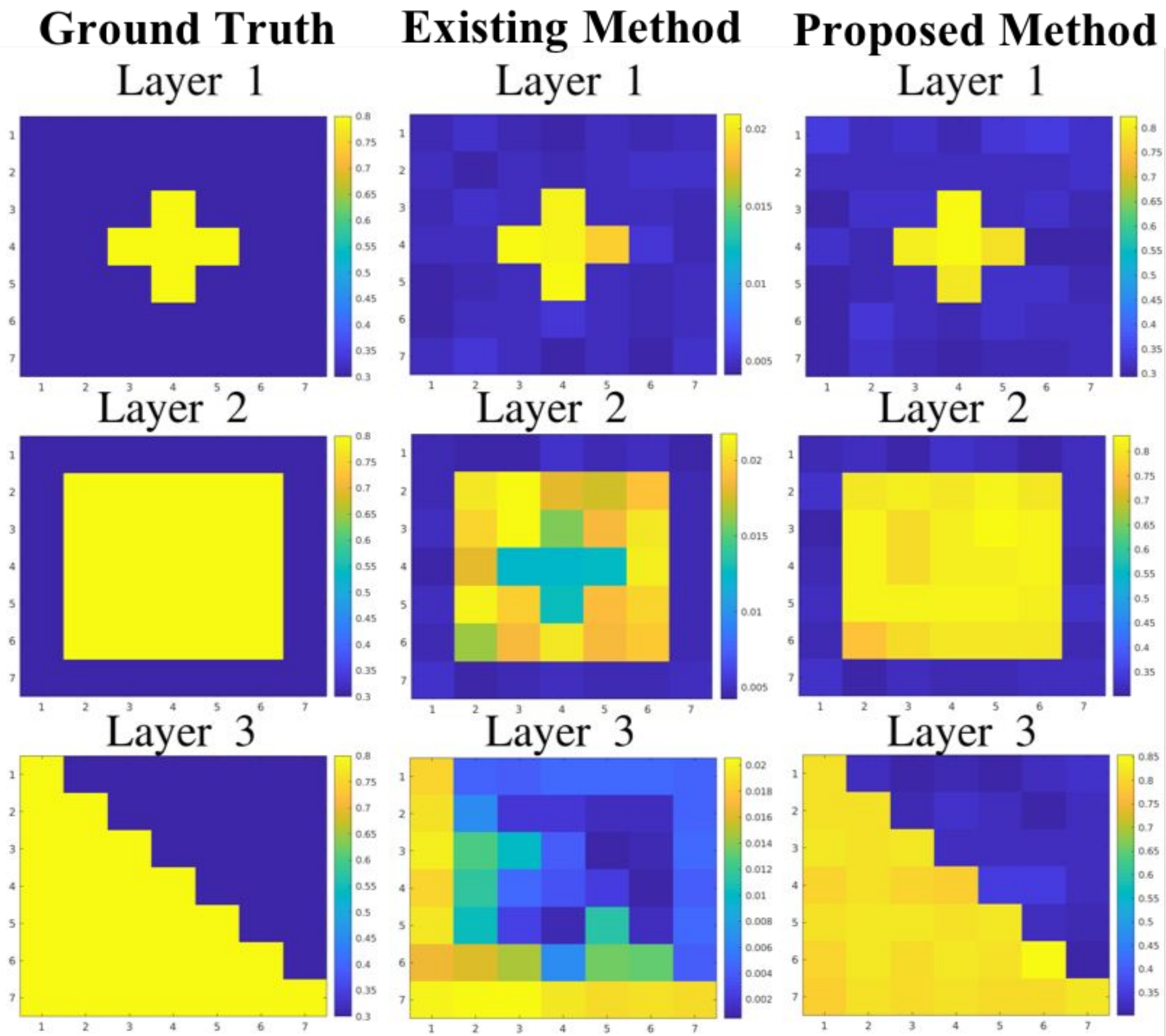
$\tau > 0$

Simulation with synthetic data (I)



Synthetic validation on a three-layer sample pixels with dielectric permittivity profiles of (a) [3; 3; 8], (b) [3; 8; 3]

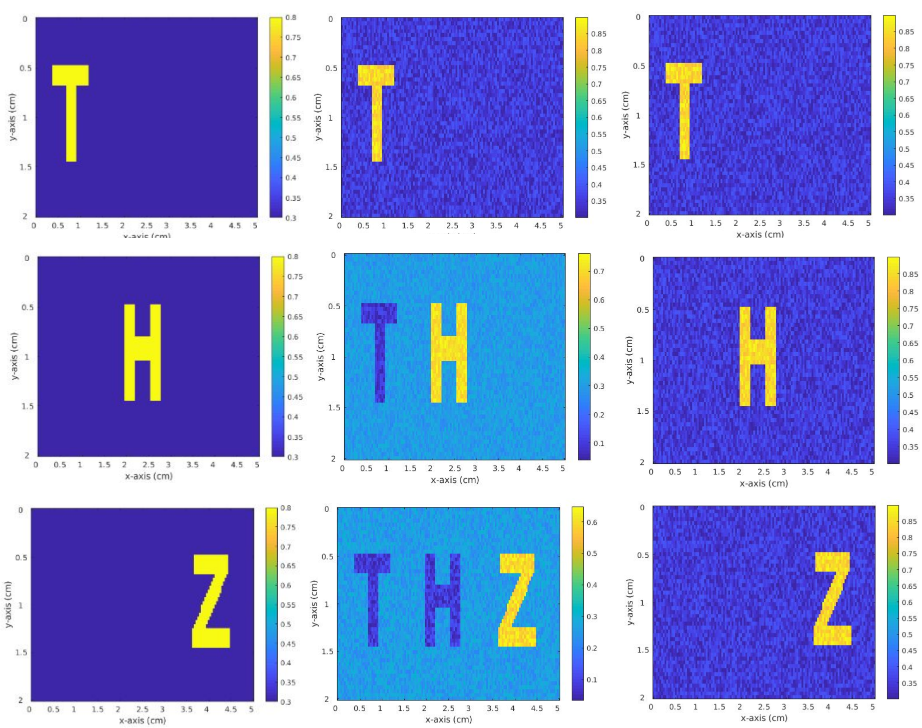
Simulation with synthetic data (II)



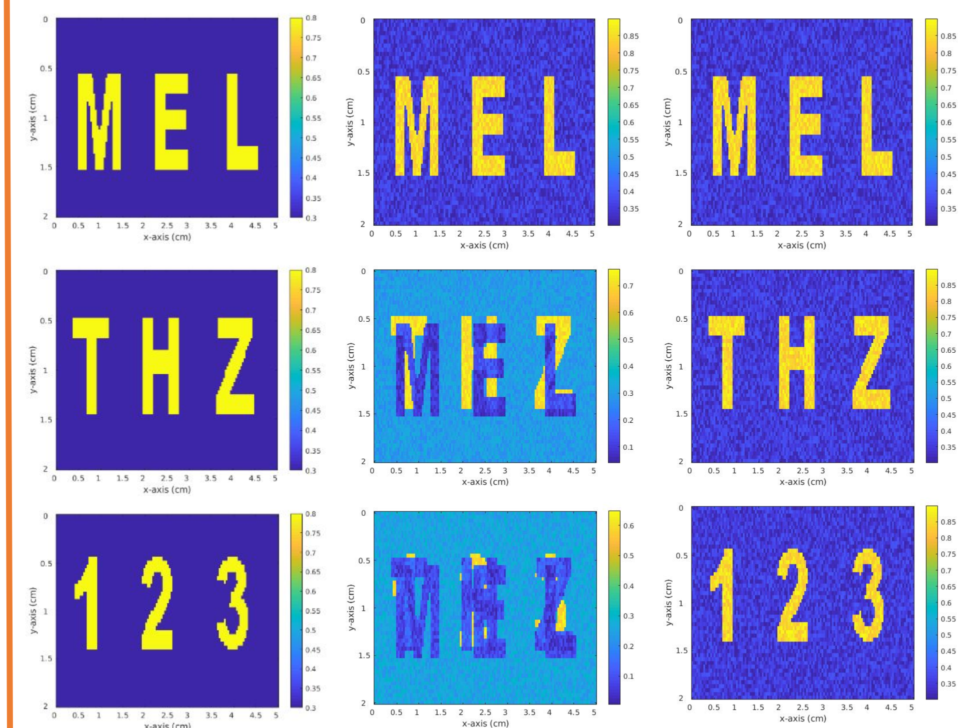
The sliced view of the layered structure: ground truth, shadow effects
and our shadow removal using our proposed method

Simulation with synthetic data (III)

Non-overlapping content



Overlapping content



The sliced view of the layered structure: ground truth, shadow effects and our shadow removal using our proposed method for non-overlapping and overlapping content with white Gaussian noise (variance 0.1)

Summary

- The shadow effect in the THz-TDS multi-layer image has been removed by using nonlinear inverse scattering model by capturing the interaction between the dielectric permittivity profile and the THz wavefield.
- The proposed method recovers the multi-layer structure by solving a 1D nonlinear inverse scattering model via an iterative and sequential optimization over frequencies.
- Numerical results shows the effectiveness of the proposed.

Thank you