A Case Study of Basic Sequential Detection Methods

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Estimation and Detection Theory

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Motivation

- Difference between fixed length and sequential detection methods

Theory of Sequential Detection

- Choose $\mathcal{H}_0$ if,

$$L(\boldsymbol{x}[n]) = \frac{p(\boldsymbol{x}[n];\mathcal{H}_1)}{p(\boldsymbol{x}[n];\mathcal{H}_0)} \leq B$$

- Choose $\mathcal{H}_1$ if,

$$L(\boldsymbol{x}[n]) = \frac{p(\boldsymbol{x}[n];\mathcal{H}_1)}{p(\boldsymbol{x}[n];\mathcal{H}_0)} \geq A$$

- Otherwise, get another sample, increase N by one, and repeat the test.

Theory of Sequential Detection (contd.)

$$P_D = \int_{\gamma}^{\infty} p(\boldsymbol{x}[n]; \mathcal{H}_0)\, d\boldsymbol{x}[n]$$

$$P_{FA} = \int_{\gamma}^{\infty} p(\boldsymbol{x}[n]; \mathcal{H}_1)\, d\boldsymbol{x}[n]$$

$$A = \frac{P_D}{P_{FA}}$$

$$B = \frac{1 - P_D}{1 - P_{FA}}$$

Numerical Analysis

- Case I: Binary communication channel

$$\mathcal{H}_0: \boldsymbol{x}[n] = \boldsymbol{w}[n] \qquad , n = 0, \ldots, N-1$$
$$\mathcal{H}_1: \boldsymbol{x}[n] = \mu + \boldsymbol{w}[n] \quad , n = 0, \ldots, N-1$$

- Case II: Radar and Active Sonar

$$\mathcal{H}_0: \boldsymbol{x}[n] = \boldsymbol{w}[n] \qquad , n = 0, \ldots, N-1$$
$$\mathcal{H}_1: \boldsymbol{x}[n] = \boldsymbol{s}[n] + \boldsymbol{w}[n] , n = 0, \ldots, N-1$$
$$\boldsymbol{s}[n] = \mu \cos(\omega \mathrm{n})$$

- Case III: Passive underwater detection of ships

$$\mathcal{H}_0: \boldsymbol{x}[n] = \boldsymbol{w}[n] \qquad , n = 0, \ldots, N-1$$
$$\mathcal{H}_1: \boldsymbol{x}[n] = \boldsymbol{s}[n] + \boldsymbol{w}[n] , n = 0, \ldots, N-1$$

Results

- Case I:

  Fixed length detection:

  - about 25 samples required for $P_{FA} = 0.1$ and $P_D = 0.9$

  Sequential detection:

  - 18 samples are needed to get a $P_{FA} = 0.1004$
  - 16 samples are needed on average to get a $P_D = 0.9240$

Results (contd.)

- Case II:

  Fixed length detection:

  - about 40 samples required for $P_{FA} = 0.1$ and $P_D = 0.9$

  Sequential detection:

  - 32 samples are needed to get a $P_{FA} = 0.085$
  - 32 samples are needed on average to get a $P_D = 0.909$

Results (contd.)

- Case III:

  Fixed length detection:

  - about 30 samples required for $P_{FA} = 0.1$ and $P_D = 0.9$

  Sequential detection:

  - 23 samples are needed to get a $P_{FA} = 0.0815$
  - 15 samples are needed on average to get a $P_D = 0.9368$

Conclusion

- The number of data samples is reduced using sequential detection.
- Sequential detection tries to get the decision rule from a single data sample. If a decision is not reached, another sample (feature) is included in the test, and so on. Eventually, the smallest possible subset of data samples that converges to a decision is attained and hence the decision is made.
- The important thing is that we rely on this smallest possible ‘decision making’ subset of data.

Thank you!

- Any Questions?