Mathematical Time Domain Study of Negative Feedback System Using Limiting Progression

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Abstract—Every stable feedback system has certain finite limiting value with respect to time. This paper describes a mathematical analytical study for stable negative feedback system with the help of limiting progressions. Some limiting progressions described in this paper have a finite limiting value which can be predicted previously using the characteristics parameters of the system by analytical method. Some of these parameters are independent and primary properties of the system itself. The final value of the feedback system having transfer function as a limiting progression can be predicted and sometimes be controlled using the parametric solution.

Index Terms—Control system, Limiting progression, Limiting progressive function, Negative feedback system, Predicting expression, Transfer function.

1 Introduction

MOSTof the negative feedback systems are stable with respect to time, as they converge to a finite limiting value. And marginally stable feedback systems are bounded-oscillatory in nature.Fig.1 describes a general configuration a negative feedback system.

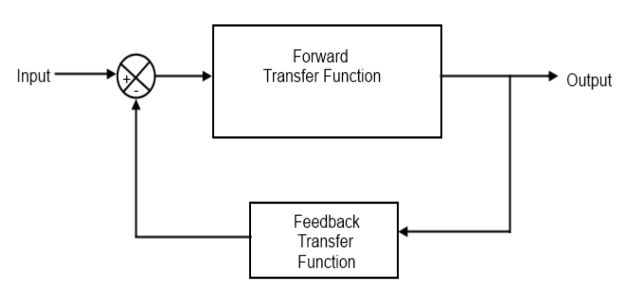

Fig.1.a. General Representation of Negative Feedback System

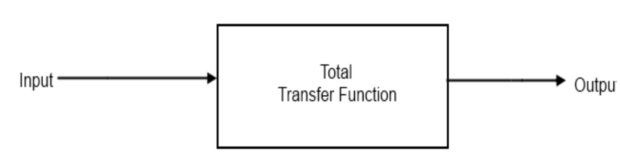

Fig.1.b. Equivalent Representation of Negative Feedback System

We know that every negative feedback system has a damping coefficient (ζ) depending which they are classified into three sections:

a. Under-damped system,
b. Critically damped system and
c. Over-damped system.

Now in this paper we will discuss about the mathematical study of these three systems with the help of Limiting Progressions.

Definition of Limiting Progressions: There are some sorts of series which are defined by a single valued iterative function (expression), where the result (dependent variable) is again used as the independent variable in the same expression and a limiting value can be reached as we go for infinite times of iteration. This type of series can be called as LIMITING PROGRESSION. So, in Limiting Progression the output is totally feedback to the input, NOT partially. Rather we can say that the output in certain state is totally put as the input of the next state.

i.e. if $f(x,i)$ is the iterative function of random variable x and i be the order of iteration and is defined as $f(x,i+1) = f(f(x,i),i+1)$ and $\lim_{i\to\infty} f(x,i) = c$ (constant), then $f(x,i)$ is called LIMITING PROGRESSIVE FUNCTION.

And the equation $u(system\ parameters)$ which predicts this constant term c is called PREDICTING EXPRESSION.

Such that $u(system\ parameters) = c$

2 Discussion about Some Types of Limiting Progressions

Let us consider some types of limiting progressions.

2.1 Type I

Limiting Progressive Function:

$$f(x,i) = (x/p + n)_i \ \ where\ x \in R, p \in R - \{0,1\}, n \in R - \{0\}, i \in I$$

Where R represents set of REAL numbers and I represents set of INTEGERS.

Predicting Expression:$u(p,n) = \frac{np}{p-1}$

Form:
$f(x,i) = \left(\frac{x}{p} + n\right)_i$ where $x \in R$ is a random variable,

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$p \in R-\{0,1\}$is a real constant,
$n \in R-\{0\}$is a real constant,
$i \in I$isthe order of iteration. … (1.1)

This is one of the limiting series. Here we begin with $x|_i$ as a random variable, then apply to (1.1). We get a result. This result is now treated as $x|_{i+1}$. So further repeating this iterative method, we will be getting a limiting value where $\lim_{i\to\infty} f(x,i) = c$(constant).

This series is an infinite series. But it has a limiting value towards the end. We can get the limiting value by considering the expression:

$$u(p,n) = \frac{np}{p-1} \qquad \dots (1.2)$$

So (1.2) does not depend on x, rather depends on p and n.
Now take an example:
Ex. 1.1: Suppose we take a random set, such as $x = 2341$, $p = 2$ and $n = 3$.

So$f(x,i) = \left(\frac{x}{p}+n\right)_i = \left(\frac{2341}{2}+3\right)_i$ for $i = 1$

From the expression(1.2) you can previously predict the limiting value of the series, which will be $\frac{3\times 2}{2-1} = 6$

Now if you take 9 digits after decimal point, the series will be:
$(2341/2+3)_1 = 1173{\cdot}5$

$(1173\cdot 5/2+3)_2 = 589{\cdot}75$

$(589\cdot 75/2+3)_3 = 297{\cdot}875$and so on. Next results will be:

151·9375	for $i = 4$
78·96875	for $i = 5$
42·484375	for $i = 6$
24·2421875	for $i = 7$
15·12109375	for $i = 8$
10·56054688	for $i = 9$
8·280273438	for $i = 10$
7·140136719	for $i = 11$
6·570068359	for $i = 12$
6·28503418	for $i = 13$
6·14251709	for $i = 14$
6·071258545	for $i = 15$
6·035629272	for $i = 16$
6·017814636	for $i = 17$
6·008907318	for $i = 18$
6·004453659	for $i = 19$
6·00222683	for $i = 20$
6·001113415	for $i = 21$
6·000556707	for $i = 22$
6·000278354	for $i = 23$
6·000139177	for $i = 24$
6·000069588	for $i = 25$
6·000034794	for $i = 26$
6·000017397	for $i = 27$
6·000008699	for $i = 28$
6·000004349	for $i = 29$
6·000002175	for $i = 30$
6·000001087	for $i = 31$
6·000000544	for $i = 32$
6·000000272	for $i = 33$
6·000000136	for $i = 34$
6·000000068	for $i = 35$
6·000000034	for $i = 36$
6·000000017	for $i = 37$
6·000000008	for $i = 38$
6·000000004	for $i = 39$
6·000000002	for $i = 40$
6·000000001	for $i = 41$
6·000000001	for $i = 42$
6·000000000	for $i = 43$
6·000000000	for $i = 44$

… value repeating
or approx. 6.

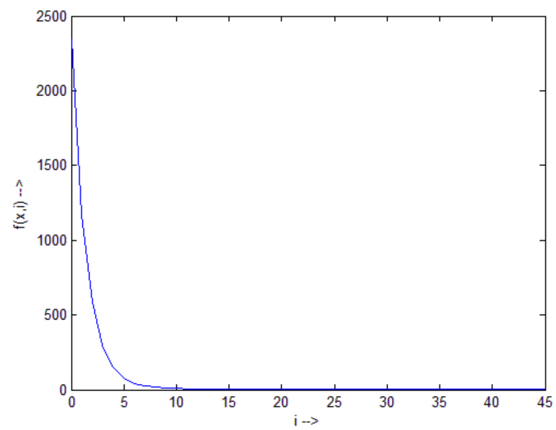

Fig.2.a. Output response of Ex.1.1

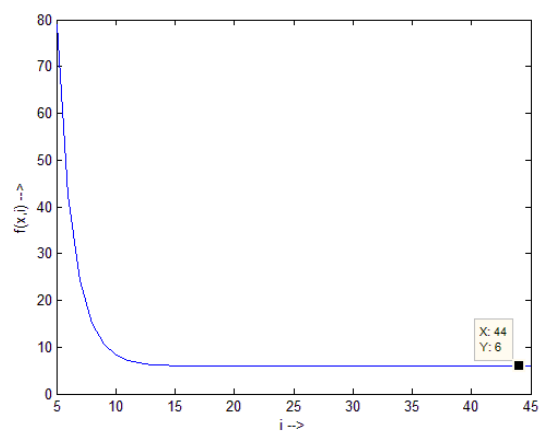

Fig.2.b. More detailed and zoomed in output response of Ex.1.1

So (1.2) is true analytically.
Ex.1.2: Now take another set for example for $f(x,i)$, $x = 123{\cdot}29$, $p = 2$ and $n = 3$ and put them into (1.1)

Here also our predicted result will be $\frac{3\times 2}{2-1} = 6$, as seen earlier.

ISSN 2229-5518

So the results will be:

$\left(\frac{123\cdot 29}{2}+3\right) = 64\cdot 645$ for $i = 1$

Next iterative results will be:

35·3225	for $i = 2$
20·66125	for $i = 3$
13·330625	for $i = 4$
9·6653125	for $i = 5$
7·83265625	for $i = 6$
6·916328125	for $i = 7$
6·458164063	for $i = 8$
6·229082031	for $i = 9$
6·114541016	for $i = 10$
6·057270508	for $i = 11$
6·028635254	for $i = 12$
6·014317627	for $i = 13$
6·007158813	for $i = 14$
6·003579407	for $i = 15$
6·001789703	for $i = 16$
6·000894852	for $i = 17$
6·000447426	for $i = 18$
6·000223713	for $i = 19$
6·000111856	for $i = 20$
6·000055928	for $i = 21$
6·000027964	for $i = 22$
6·000013982	for $i = 23$
6·000006991	for $i = 24$
6·000003496	for $i = 25$
6·000001748	for $i = 26$
6·000000874	for $i = 27$
6·000000437	for $i = 28$
6·000000218	for $i = 29$
6·000000109	for $i = 30$
6·000000055	for $i = 31$
6·000000027	for $i = 32$
6·000000014	for $i = 33$
6·000000007	for $i = 34$
6·000000003	for $i = 35$
6·000000002	for $i = 36$
6·000000001	for $i = 37$
6·000000000	for $i = 38$
6·000000000	for $i = 39$

…value repeating
or approx. 6.

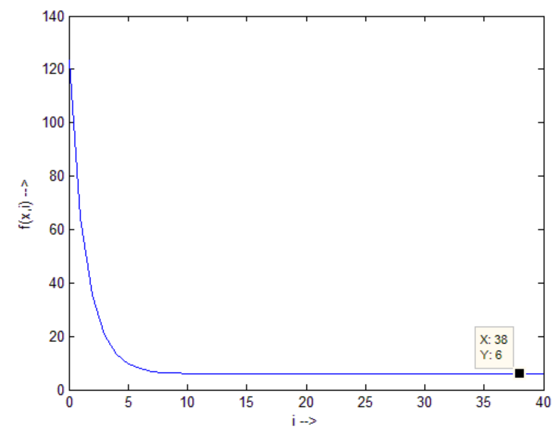

Fig.3. Output response of Ex.1.2

So (1.2) is true analytically whatever x may be.

Ex.1.3: Now we take another example:

$f(x,i) = \left(\frac{x}{p}+n\right)_i$ for $x = 1523, p = 121, n = 45$

The prediction for answer is: $\frac{45\times 121}{121-1} = 45\cdot 375$

Now let's see, $(1523/121+45)_i$
The iterative results are:

57·58677686	for $i = 1$
45·47592378	for $i = 2$
45·37583408	for $i = 3$
45·37500689	for $i = 4$
45·37500006	for $i = 5$
45·375	for $i = 6$
45·375	for $i = 7$

…value repeating

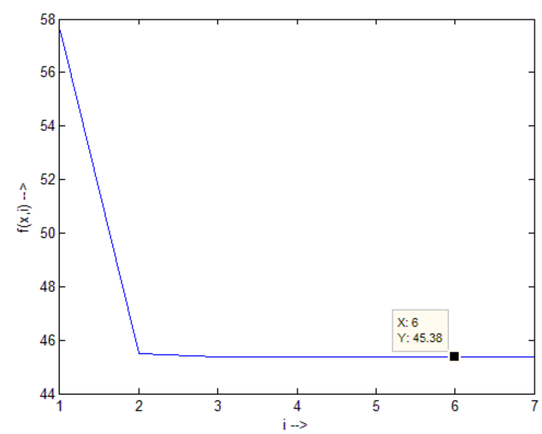

Fig.4. Output response of Ex.1.3

Ex.1.4: Now we are taking another different example:

$f(x,i) = \left(\frac{x}{p} + n\right)_i$ for $x = 189{\cdot}34, p = -19, n = -6$

Predicted answer is: $\frac{(-6)\times(-19)}{(-19-1)} = -5{\cdot}7$

Now let's see, $\left(\frac{189{\cdot}34}{-19}\right) - 6$

The iterative results are:

–15·96526316	for $i = 1$
–5·159722992	for $i = 2$
–5·728435632	for $i = 3$
–5·698503388	for $i = 4$
–5·700078769	for $i = 5$
–5·699995854	for $i = 6$
–5·700000218	for $i = 7$
–5·699999989	for $i = 8$
–5·700000001	for $i = 9$
–5·7	for $i = 10$
–5·7	for $i = 11$

… value repeating

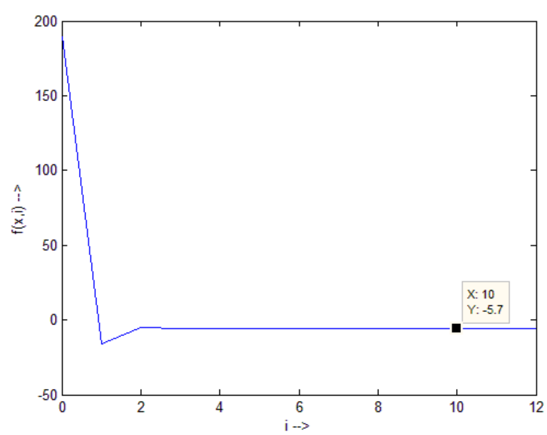

Fig.5. Output response of Ex.1.4

Then our prediction was right and exact in all the cases.
Some limiting values for real variable p :
Take p be 1,2,3 and so on and n be another real variable.
Then the results will be:

for $p = 2 \to 2 \times n \times \frac{1}{1}$ ['→' indicates the limiting value]

for $p = 3 \to 3 \times n \times \frac{1}{2}$

for $p = 4 \to 4 \times n \times \frac{1}{3}$

for $p = 5 \to 5 \times n \times \frac{1}{4}$

for $p = 6 \to 6 \times n \times \frac{1}{5}$

for $p = 7 \to 7 \times n \times \frac{1}{6}$

for $p = 8 \to 8 \times n \times \frac{1}{7}$

for $p = 9 \to 9 \times n \times \frac{1}{8}$

for $p = 10 \to 10 \times n \times \frac{1}{9}$ and so on…

Here $p = 1$ is not allowed as the term $\frac{1}{1-1} = \frac{1}{0}$ is not allowed.

So, for $f(x,i)$ be the iterative function of random variable x and i be the order of iteration and is defined as $f(x,i+1) = f(f(x,i),i+1)$, then $\lim_{i\to\infty} f(x,i) =$ constant.
So this is an example of limiting progression.

2.2 Type II

Limiting Progressive Function:

$$f(x,i) = \left(p \times (x)^{\frac{1}{n}}\right)_i \; where x \in R - \{0\}, p \in R - \{0\}, n \in R - [-1,+1], i \in I$$

Predicting Expression:

$$u(p,n) = \pm(|p|)^{\frac{n}{n-1}}$$

Form:

$f(x,i) = \left(p \times (x)^{\frac{1}{n}}\right)_i$ where $x \in R - \{0\}$ is a random variable,

$p \in R - \{0\}$ is a real constant,
$n \in R - [-1,+1]$ is a real constant,
$i \in I$ is the order of iteration. … (2.1)

Here also we begin with x as a random variable, then apply it to (2.1). We get a value of $f(x,i)$. This result now will be treated as next x. Further we are going to apply it to (2.1) and so on and so forth.

This series also has no end; rather it has a limiting value. We can get the limiting value by considering the following expression:

$\mathrm{u}(\mathrm{p},\mathrm{n}) = \pm(|p|)^{\frac{n}{n-1}}$ … (2.2)

So (2.2) does not depend on x, rather depends on p and n and sign of x.
Let's consider different cases:

Case (1): $x > 0, p > 0, n > 1$
In this case the limiting value will be:

$u(p,n) = (p)^{\frac{n}{n-1}}$ … (2.3)

Let's take some examples.

Ex.2.1: Initialize $f(x,i) = \left(p \times (x)^{\frac{1}{n}}\right)_i$ for $x = 23, p = 2, n = 3$

So predicted answer is:

$$(2)^{\frac{3}{3-1}} = 2^{\frac{3}{2}} = 2{\cdot}828427125$$

Now let's calculate $f(x,i)$ for different values of i.
The iterative results will be:

5·68773396	$for\ i = 1$
3·570067442	$for\ i = 2$
3·056718667	$for\ i = 3$
2·902564095	$for\ i = 4$
2·852926624	$for\ i = 5$
2·836570158	$for\ i = 6$
2·831138868	$for\ i = 7$
2·829330751	$for\ i = 8$
2·828728301	$for\ i = 9$
2·828527513	$for\ i = 10$
2·828460587	$for\ i = 11$
2·828438279	$for\ i = 12$
2·828430843	$for\ i = 13$
2·828428364	$for\ i = 14$
2·828427538	$for\ i = 15$
2·828427262	$for\ i = 16$
2·828427171	$for\ i = 17$
2·82842714	$for\ i = 18$
2·82842713	$for\ i = 19$
2·828427126	$for\ i = 20$
2·828427125	$for\ i = 21$
2·828427125	$for\ i = 22$

…value repeating

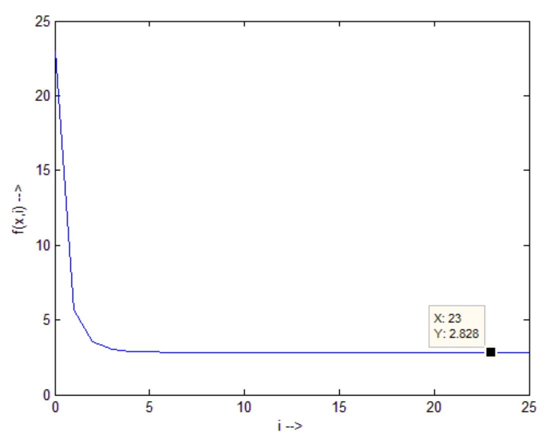

Fig.6. Output response of Ex.2.1

So (2.3) is proved analytically.
If we take $x = 1345, p = 2, n = 3$
Here also the limiting value will be 2·828427125 $\quad for\ i = 22$.
So (2.3) is true always for a single set of $\{p, n\}$.

Ex.2.2: Now let's take another example:
Take x=27·345, p=12·81, n=2·9
Our predicted answer will be:

$$(12.81)^{\left(\frac{2.9}{2.9-1}\right)} = (12.81)^{\left(\frac{2.9}{1.9}\right)} = 49.0308841$$

Now let's calculate $f(x,i)$ for different values of i.
The iterative results will be:

40·08891269	$for\ i = 1$
45·74210241	$for\ i = 2$
47·87093724	$for\ i = 3$
48·62776152	$for\ i = 4$
48·89150021	$for\ i = 5$
48·98277586	$for\ i = 6$
49·01428972	$for\ i = 7$
49·02516127	$for\ i = 8$
49·02891064	$for\ i = 9$
49·03020359	$for\ i = 10$
49·03064944	$for\ i = 11$
49·03080319	$for\ i = 12$
49·0308562	$for\ i = 13$
49·03087448	$for\ i = 14$
49·03088079	$for\ i = 15$
49·03088296	$for\ i = 16$
49·03088371	$for\ i = 17$
49·03088397	$for\ i = 18$
49·03088406	$for\ i = 19$
49·03088409	$for\ i = 20$
49·0308841	$for\ i = 21$
49·0308841	$for\ i = 22$

…value repeating

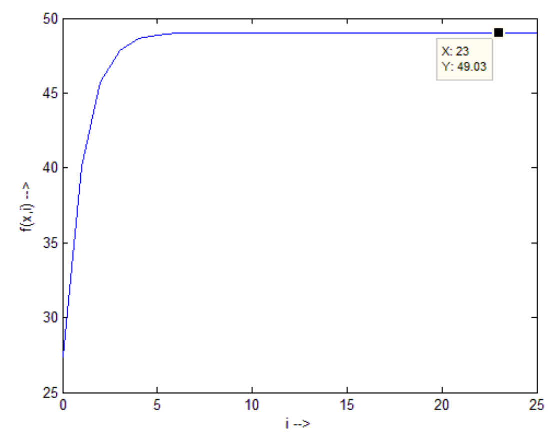

Fig.7. Output response of Ex.2.2

So (2.3) is proved analytically.

Case (2): $x < 0, p > 0, n\ odd\ integer$
In this case the limiting value will be:

$$u(p,n) = -(p)^{\frac{n}{n-1}} \qquad \ldots (2.4)$$

Ex.2.3: Take an example: $x = -12 \cdot 67, p = 3.5, n = 3$ (odd taken).
Now the expression will be:

$$f(x,i) = \left(3.5 \times (-12.67)^{\frac{1}{3}}\right)_i$$

Predicted answer: $-(3\cdot 5)^{3/2} = -6\cdot 547900427$

The iterative results will be:

–8·159438048	$for\ i = 1$
–7·04619721	$for\ i = 2$
–6·709955537	$for\ i = 3$
–6·601479188	$for\ i = 4$
–6·565711522	$for\ i = 5$
–6·553832083	$for\ i = 6$
–6·549877049	$for\ i = 7$
–6·548559235	$for\ i = 8$
–6·548120022	$for\ i = 9$
–6·547973624	$for\ i = 10$
–6·547924826	$for\ i = 11$
–6·54790856	$for\ i = 12$
–6·547903138	$for\ i = 13$
–6·54790133	$for\ i = 14$
–6·547900728	$for\ i = 15$
–6·547900527	$for\ i = 16$
–6·54790046	$for\ i = 17$
–6·547900438	$for\ i = 18$
–6·547900431	$for\ i = 19$
–6·547900428	$for\ i = 20$
–6·547900427	$for\ i = 21$
–6·547900427	$for\ i = 22$

…value repeating

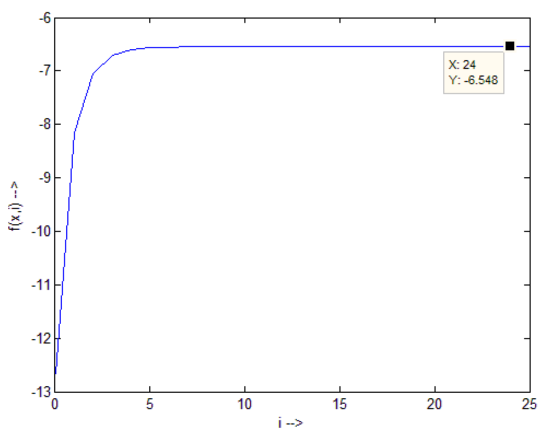

Fig.8. Output response of Ex.2.3

So (2.4) is proved analytically.

Ex.2.4: Take another example: $x = -12\cdot 67, p = 3.5, n = 2$ (even taken).
So the expression will be:

$$\left(3.5 \times (-12.67)^{\frac{1}{2}}\right)_i \text{ is not defined.}$$

Case (3):$p < 0, n\ odd\ integer$
Here the limiting value should be:

$$u(p,n) = \pm|p|^{\frac{n}{n-1}} \ldots (2.5)$$

In this case the positive and negative limiting values will gradually come one after one and the system will be oscillatory.
Ex.2.5: Take an example: $x = 2, p = -2, n = 3$
Predicted answer is:

$$\pm|-2|^{\frac{3}{2}} = \pm 2.828427125$$

Now the iterative results for $f(x,i)$will be as follows:

–2·5198421	$for\ i = 1$
2·72158	$for\ i = 2$
–2·792353286	$for\ i = 3$
2·816351026	$for\ i = 4$
–2·824396016	$for\ i = 5$
2·827082783	$for\ i = 6$
–2·82797894	$for\ i = 7$
2·828277722	$for\ i = 8$
–2·828377323	$for\ i = 9$
2·828410524	$for\ i = 10$
–2·828421591	$for\ i = 11$
2·82842528	$for\ i = 12$
–2·82842651	$for\ i = 13$
2·82842692	$for\ i = 14$
–2·828427056	$for\ i = 15$
2·828427102	$for\ i = 16$
–2·828427117	$for\ i = 17$
2·828427122	$for\ i = 18$
–2·828427124	$for\ i = 19$
2·828427124	$for\ i = 20$
–2·828427125	$for\ i = 21$
2·828427125	$for\ i = 22$
–2·828427125	$for\ i = 23$
2·828427125	$for\ i = 24$

…value repeating

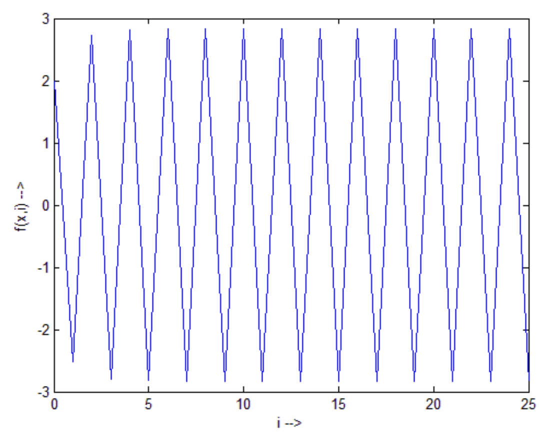

Fig.9. Output response of Ex.2.5

So (2.4) is proved analytically.

Case (4): $n < 0$
In this case the predicted answer will be as (2·3) i.e.

$$u(p,n) = (p)^{\frac{n}{n-1}}$$

Ex.2.6: Example: $x = 2, p = 2, n = -3$
Predicted answer:

$$(2)^{\frac{-3}{-3-1}} = (2)^{\frac{-3}{-4}} = (2)^{\frac{3}{4}} = 1.681792831$$

And the iterative results for $f(x,i)$will be:

1·587401052	$for\ i = 1$
1·714487966	$for\ i = 2$
1·671033598	$for\ i = 3$
1·685394614	$for\ i = 4$
1·680593947	$for\ i = 5$
1·682192648	$for\ i = 6$
1·681659579	$for\ i = 7$
1·68183725	$for\ i = 8$
1·681778024	$for\ i = 9$
1·681797766	$for\ i = 10$
1·681791185	$for\ i = 11$
1·681793379	$for\ i = 12$
1·681792648	$for\ i = 13$
1·681792891	$for\ i = 14$
1·68179281	$for\ i = 15$
1·681792837	$for\ i = 16$
1·681792828	$for\ i = 17$
1·681792831	$for\ i = 18$
1·68179283	$for\ i = 19$
1·681792831	$for\ i = 20$
1·68179283	$for\ i = 21$
1·681792831	$for\ i = 22$
1·681792831	$for\ i = 23$
1·681792831	$for\ i = 24$

…value repeating

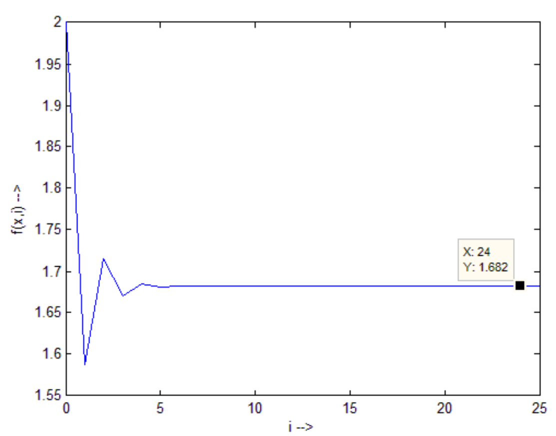

Fig.10. Output response of Ex.2.6

So prediction in (2.3) is also proved analytically.

But in case of $p = 0, n = \pm 1$ this series cannot be obtained.

So, for $f(x,i)$ be the iterative function of random variable x and i be the order of iteration and is defined as $f(x,i+1) = f(f(x,i),i+1)$, then$\lim_{i\to\infty} f(x,i) =$ constant.
So this is also an example of limiting progression.

2.3 Type III

Limiting Progressive Function:
$f(x,i) = (\cos(x))_i\ where\ x \in (-\infty,+\infty), i \in I$
Predicting Expression:
$u = 0·9998477415310881129598107686798$

Form:
$f(x,i) = (\cos(x))_i$where $x \in (-\infty,+\infty)$and x is in degree,
$i \in I$ is the order of iteration. … (3.1)
In this case also if we consider x as a random variable like all the previous types, we get a result from (3.1), then the answer again be considered as x and so on. In this case, this series will have a limiting value, which is 0·9998477415310881129598107686798 if we consider 31 digits after decimal point. So,
$\lim_{i\to\infty} f(x,i) =$ 0·9998477415310881129598107686798 (constant)

Ex.3.1: Now take an example: suppose we take any random variable $x = 23$.
Then the iterative results will be:

0·9205048534524403273968947233004 6	$for\ i = 1$
0·9998709471608107881384803433285 1	$for\ i = 2$
0·9998477344636020595343728637747 9	$for\ i = 3$
0·9998477415332405552748833631971 6	$for\ i = 4$
0·9998477415310874574214904847308 4	$for\ i = 5$
0·9998477415310881131594586228178 8	$for\ i = 6$
0·9998477415310881129597499648115 5	$for\ i = 7$
0·9998477415310881129598107871979 6	$for\ i = 8$
0·9998477415310881129598107686741 6	$for\ i = 9$
0·9998477415310881129598107686798	$for\ i = 10$
0·9998477415310881129598107686798	$for\ i = 11$

…value repeating

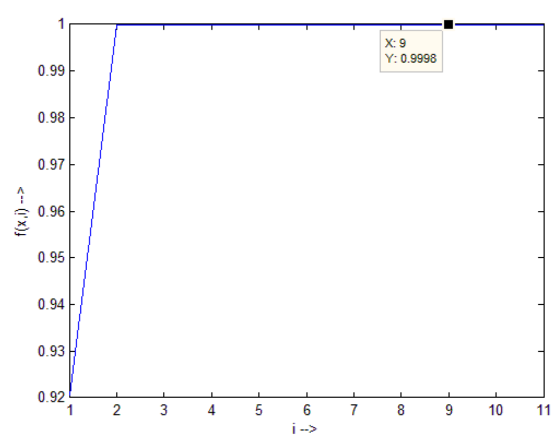

Fig.11. Output response of Ex.3.1

Ex.3.2: Take another example: $x = 2645$
Then the iterative results will be:
–0·57357643635104609610803191282616 $for\ i = 1$
0·99994989238691479899521937732222 $for\ i = 2$
0·9998477104188857772677217602915 $for\ i = 3$
0·99984774154056350767740513976426 $for\ i = 4$
0·99984774153108522717546536452938 $for\ i = 5$
0·99984774153108811383869249723479 $for\ i = 6$
0·99984774153108811295954310034415 $for\ i = 7$
0·99984774153108811295981085019969 $for\ i = 8$
0·99984774153108811295981076865497 $for\ i = 9$
0·99984774153108811295981076867981 $for\ i = 10$
0·9998477415310881129598107686798 $for\ i = 11$
0·9998477415310881129598107686798 $for\ i = 12$
…value repeating

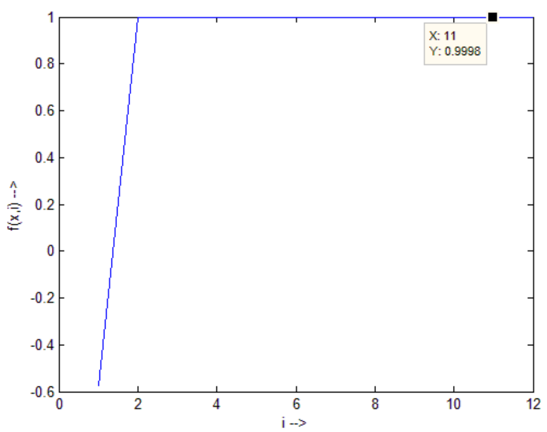

Fig.12. Output response of Ex.3.2

Ex.3.3: Take another example: $x = -67$
The iterative results will be:
0·39073112848927375506208458888909 $for\ i = 1$
0·99997674699528153455312175786626 $for\ i = 2$
0·99984770223921958855808301788233 $for\ i = 3$
0·99984774154305467057989511316751 $for\ i = 4$
0·99984774153108446847789970519695 $for\ i = 5$
0·99984774153108811406975807530513 $for\ i = 6$
0·99984774153108811295947272803272 $for\ i = 7$
0·99984774153108811295981087163197 $for\ i = 8$
0·99984774153108811295981076864845 $for\ i = 9$
0·99984774153108811295981076867981 $for\ i = 10$
0·9998477415310881129598107686798 $for\ i = 11$
0·9998477415310881129598107686798 $for\ i = 12$
…value repeating

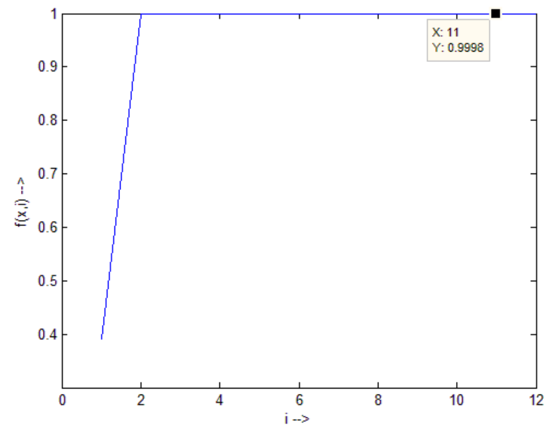

Fig.13. Output response of Ex.3.3

Then our prediction was exactly right. This is proved now by analytical method.
So, as we can see
$\cos(0{\cdot}9998477415310881129598107686798)$
$= 0{\cdot}9998477415310881129598107686798$
In this case we can say $\cos(x) = x$
where x = 0·9998477415310881129598107686798
This type of series is not applicable for $\sin x, \tan x$, or other trigonometric expression.
So, for $f(x,i)$ be the iterative function of random variable x and i be the order of iteration and is defined as $f(x,i+1) = f(f(x,i), i+1)$, then$\lim_{i\to\infty} f(x,i) =$ constant.
So this is also an example of limiting progression.

2.4 Type IV

Limiting Progressive Function:
$f(x,i) = (\tan^{-1} x)_i\ where\ x \in (-\infty, +\infty) - \{0\}, i \in I$
Predicting Expression:
$$u = \pm 89{\cdot}35883917$$

Form:
$f(x,i) = (\tan^{-1} x)_i\ where\ x \in (-\infty, +\infty) - \{0\}$ is a value, not in degree, $i \in I$ is the order of iteration. … (4.1)
In this case also if we consider x as a random variable, we get a result from (4.1) for $i - th$ iteration, then this value is again used as x for $i + 1 - st$ iteration and so on. In this case also this series will have a limiting value which is $\pm$89·35883917 if we consider 8 digits after point. If x be positive, the limiting value will be positive and if x be negative, the limiting value will be negative.

Ex.4.1: Now let's take an example: $x = 45$
Then the iterative results will be:
88·72696998 $for\ i = 1$
89·35427352 $for\ i = 2$
89·35880641 $for\ i = 3$
89·35883893 $for\ i = 4$
89·35883916 $for\ i = 5$

89·35883917 $for\ i = 6$
89·35883917 $for\ i = 7$
…value repeating.

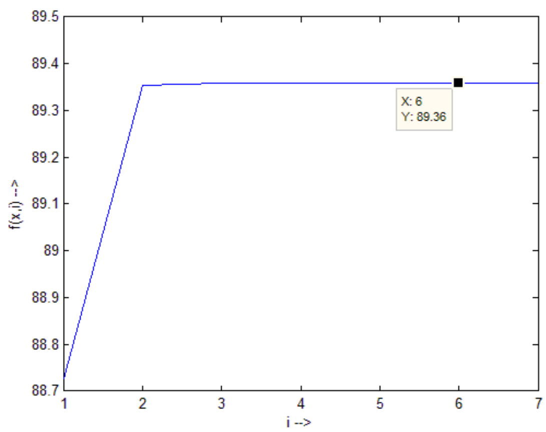

Fig.14. Output response of Ex.4.1

Ex.4.2: Take another example: $x = 23134$
Then the iterative results will be:
89·99752331 $for\ i = 1$
89·36338891 $for\ i = 2$
89·35887181 $for\ i = 3$
89·3588394 $for\ i = 4$
89·35883917 $for\ i = 5$
89·35883917 $for\ i = 6$
…value repeating.

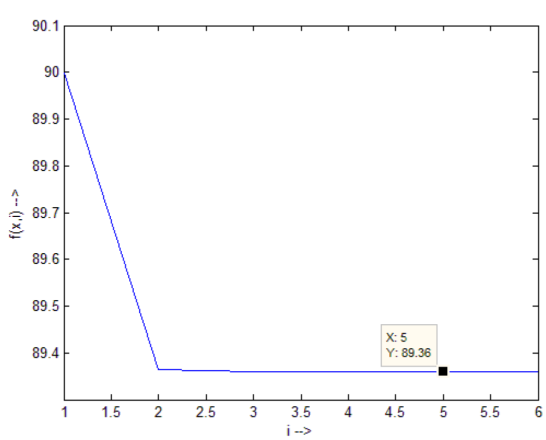

Fig.15. Output response of Ex.4.2

Ex.4.3: Take another example: $x = -23134$
Then the sequential results will be:
–89·9975331 $for\ i = 1$
–89·36338891 $for\ i = 2$
–89·35887181 $for\ i = 3$
–89·3588394 $for\ i = 4$
–89·35883917 $for\ i = 5$
–89·35883917 $for\ i = 6$
…value repeating

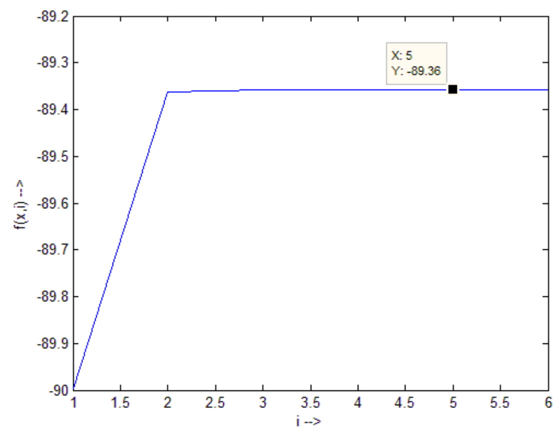

Fig.16. Output response of Ex.4.3

So, this prediction was true.
As we can see $\tan^{-1}(\pm 89{\cdot}35883917) = \pm 89{\cdot}35883917$
So, we can say $\tan^{-1}(x) = x$ where $x = \pm 89{\cdot}35883917$
This type of series is not applicable for $\sin^{-1} x$, $\cos^{-1} x$, or other inverse trigonometric expression.
So, for $f(x,i)$ be the iterative function of random variable x and i be the order of iteration and is defined as $f(x,i+1) = f(f(x,i), i+1)$, then$\lim_{i\to\infty} f(x,i) = \text{constant}$.
So this is also an example of limiting progression.

2.5 Type V

Limiting Progressive Function:

$$f(x,i) = \left(\frac{\sin x}{x}\right)_i \ where\ x \in R - \{0\}, i \in I$$

Predicting Expression:
$u = 0{\cdot}017453292$

Limiting Progressive Function:

$$f(x,i) = \left(\frac{\tan x}{x}\right)_i \ where\ x \in R - \{0\}, i \in I$$

Predicting Expression:
$$u = 0{\cdot}017453293$$

Form(I):
$f(x,i) = \left(\frac{\sin x}{x}\right)_i$ where $x \in R - \{0\}$,

$i \in I$ is the order of iteration. … (5.1)
In this case also if we consider x as a random variable, we get a result from (5.1) for $i - th$ iteration, then this value is again used as x for $i + 1 - st$ iteration and so on. In this case also this series will have a limiting value which is 0·017453292 if we consider 9 digits after decimal point.
Ex.5.1.1: Now consider an example: Let's take $x = 123$
The successive iterative results will be:
0·006818459 $for\ i = 1$

ISSN 2229-5518

0·017453292 $for\ i = 2$
0·017453292 $for\ i = 3$
and thus so on.

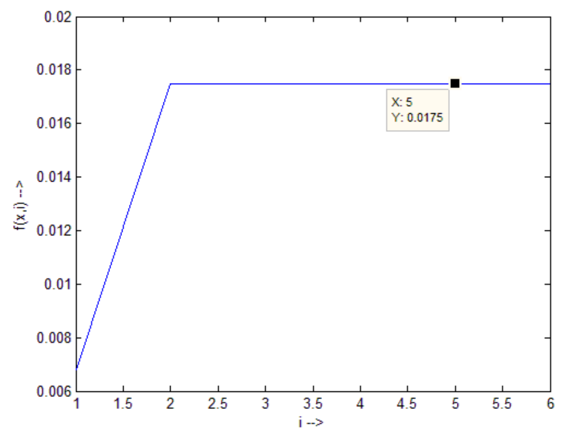

Fig.17. Output response of Ex.5.1.1

So, it is now clear that, this series has a limiting value: 0·017453292.

Ex.5.1.2: Now let's take another example: $x = -0{\cdot}785$
The iterative results will be:
0·017452746 $for\ i = 1$
0·017453292 $for\ i = 2$
0·017453292 $for\ i = 3$
………

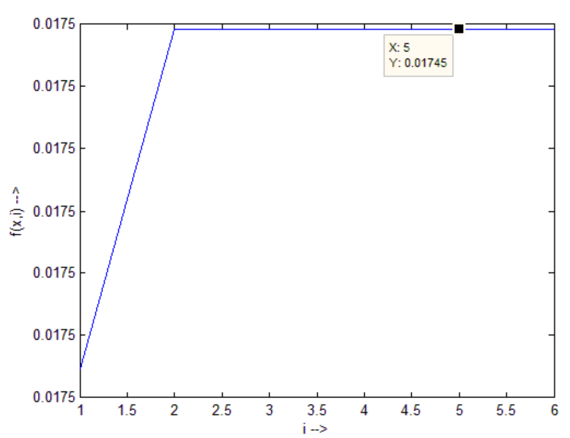

Fig.18. Output response of Ex.5.1.2

Here also the limiting value is 0·017453292.
Actually this value is: 0·017453292250022980737699843973 considering 31 digits after decimal point.

Form (II):

$$f(x,i) = \left(\frac{\tan x}{x}\right)_i \text{where } x \in R - \{0\},$$

$i \in I$ is the order of iteration. … (5.2)

In this case also if we consider x as a random variable, we get a result from (5.2) for $i - th$ iteration, then this value is again used as x for $i + 1 - st$ iteration and so on. In this case also this series will have a limiting value which is 0·017453293 if you consider 9 digits after point.

Ex.5.2.1: Now consider an example: Let's take $x = 123$
The successive results will be:=
–0·012519227 $for\ i = 1$
0·017453292 $for\ i = 2$
0·017453293 $for\ i = 3$
and thus so on.

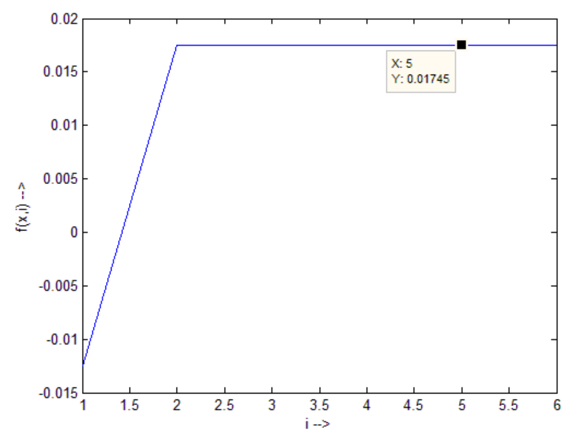

Fig.19. Output response of Ex.5.2.1

So, it is now clear that, this series has a limiting value 0·017453293.

Ex.5.2.2: Now let's take another example: $x = -0{\cdot}785$
The iterative results will be:
0·017454384 $for\ i = 1$
0·017453293 $for\ i = 2$
0·017453293 $for\ i = 3$
………

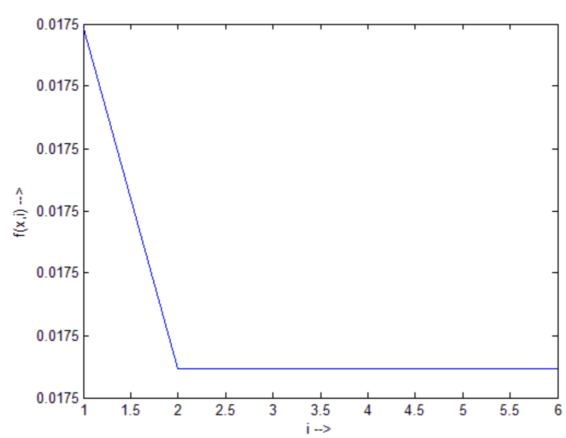

Fig.20. Output response of Ex.5.2.2

Here also the limiting value is 0·017453293.
Actually this value is: 0·0174532930597839984668346897 98considering 31 digits after decimal point. However, we have seen than these limiting values of those two series are about same (up to 8 digits after decimal point).

Moreover $\left(\frac{\cos x}{x}\right)$series has no such property.

So, for $f(x,i)$ be the iterative function of random variable x and i be the order of iteration and is defined as $f(x,i+1) = f(f(x,i),i+1)$,then$\lim_{i\to\infty} f(x,i) = \text{constant}$.
So these are also the examples of limiting progression.
There are many other types of limiting progression.

3 Application of Limiting Progression in the Context of Transfer Function of a Feedback System

For type I: $f(x,i) = (x/p + n)_i\ where\ x \in R,\ \ p \in R - \{0,1\},\ n \in R - \{0\},\ i \in I$
In reference with Ex.1.1: $x|_{i=0} = 2341,\ p = 2$ and $n = 3$.
Here $x|_{i=0}$ is the initial input and $x|_i$ are the consecutive inputs and $f(x,i)$ is the i-th output of the system. Again p and n are the independent parameters of the feedback system. Here the system is called the feedback system as the output is fully or partially used as input to the same system. $f(x,i)$is the transfer function of the feedback system.
Here from the iterative outputs of the system, it is clear that the limiting value (final value) does not depend on the input or the output of the system rather depends on the parameters of the system. So if we have only $p$and n, we will have the final value of the system, whatever be the input of the system. So we can predict the system. In these two cases the predicted value is always 6.
We have another output parameter of the system which is i. $i$is equivalent to 'time' in time domain analysis of the system. Here the value of i where the limiting value is attained, is not only depends on p and n but also the $x|_{i=0}$ i.e. the input of the system. In reference with Ex.1.1 and Ex. 1.2, the value of p and n are same in both case i.e. 2 and 3 respectively. But while reaching the limiting value, the value of i is 44 and 39 in the two different cases respectively, where the inputs were 2341 and 123.29 respectively. Hence we can conclude that the value of i also depends on initial input of the system.

For type II: $f(x,i) = \left(p \times (x)^{\frac{1}{n}}\right)_i\ \ where\ x \in R - \{0\}, p \in R - \{0\}, n \in R - [-1,+1], i \in I$

Case (1): In reference with Ex.2.1: $x|_{i=0} = 23, p = 2, n = 3$
In this case also, we can conclude in the same way through the essence of the previous discussion, that p and n are acting as independent parameters of the feedback system and i is acting as dependent parameter which is dependent on p, n and $x|_{i=0}$. As in Ex.2.1 we have discussed using different values of $x|_{i=0}$ viz. 23 and 1345, but in both cases, $p = 2, n = 3$. We have seen that the limiting value of the progression is 2·828427125 in both cases.
Case (2): Here n has a limitation while $x < 0$, it should be odd always, because j −th root of a negative real number is always imaginary when j is even integer. The rest of the conclusion is same.
Case (3): When $p < 0$, the feedback system will have special characteristics. The system will be oscillatory in nature, but not like positive feedback. The limiting value will have the same amplitude both in positive and negative side of i-axis (time axis). So this system will have marginal stability (much like sinusoidal in nature). As in Ex.2.5$x|_{i=0} = 2,\ p = -2, n = 3$, the limiting value is ±2·828427125.
Case (4): When $n < 0$, a single limiting value is attained, from both side of i-axis. Initially the system will have a decaying oscillating nature, and gradually it will attain a limiting value. As described in Ex.2.6. $x|_{i=0} = 2,\ p = 2, n = -3$, the system will attain a limiting value of 1·681792831 after passing through an initial decaying oscillation.

For type III: $f(x,i) = (\cos(x))_i\ where\ x \in (-\infty,+\infty), i \in I,$

typeIV: $f(x,i) = (\tan^{-1} x)_i\ where\ x \in (-\infty,+\infty) - \{0\}, i \in I$
andtype V: $f(x,i) = \left(\frac{\sin x}{x}\right)_i\ where\ x \in R - \{0\}, i \in I$

$$f(x,i) = \left(\frac{\tan x}{x}\right)_i\ where\ x \in R - \{0\}, i \in I$$

there is no independent parameter of these types of feedback systems. $i$is only acting as dependent parameter which is dependent only on $x|_{i=0}$ i.e. the initial value of x. These are all the examples of constant limiting progression, as whatever be the initial input condition, the final output of the system is always same. So we can't control the output of these systems.

4 Conclusions and Future Works

Prediction of output and controlling of a system based on the prediction can be done using the mathematical study of the system. This study is not complete yet, as the expression of i where the limiting value is attained is not clear yet. As i is dependent on the system parameter and the initial input value, so it must have an expression. Working on these expressions requires time domain analysis of the system. So this paper gives an initial concept and steps towards the mathematical study. All the software simulations are done in MATLAB.

Acknowledgment

I wish to thank my college Future Institute of Engineering and Management for inspiring and encouraging me as an undergraduate engineering student.

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