Ambiguity Function Shaping in FMCW Automotive Radar

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Motivation

- We study the ambiguity function shaping in frequency-modulated continuous wave (FMCW) automotive radar.
- Motivated by mitigating interference in automotive radar, we devise a low-complexity algorithm based on power-method-like iterations to minimize the ambiguity function in the range-Doppler bins corresponding to echoes from clutters in the environment.
- Shaping radar ambiguity functions has long been considered difficult from a pure design or computational perspective due to the fact that the two-dimensional nature of the ambiguity function implies the number of design constraints would grow much faster than the design variables and that the design objective (to be optimized) has a **quartic** nature.
- A cyclic iterative algorithm is introduced that recasts the quartic problem as a unimodular quadratic problem (UQP) which can be tackled using **power-method-like iterations (PMLI)**.

Problem Formulation

The transmit signal with an intra-pulse code length N can be represented as

$$s(t) = \sum_{n=1}^{N} x_n u(t - nT_c),\ 0 \le t \le T_c \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_N] \in \mathbb{C}^N$ is the slow-time sequence and the chirp is

$$u(t) = \frac{1}{\sqrt{T_c}} \exp(\mathrm{j}(2\pi f_c t + \pi K t^2)) \frac{t}{T_c}, \tag{2}$$

where $K = \frac{B}{T_c}$ is the chirp rate.
The **ambiguity function(AF)** is defined as

$$\chi(\tau, \nu) = \int_{-\infty}^{\infty} s(t) s^*(t - \tau) \exp(-\mathrm{j}2\pi\nu(t - \tau))\, dt$$

We discretize the AF, by setting $\tau = kT_c$ for $k = -N+1, \cdots, 0, \cdots, N-1$ and $\nu = \frac{p}{NT_c}$ for $p = -\frac{N}{2}, \cdots, \frac{N}{2} - 1$ for even p or $p = -\frac{N-1}{2}, \cdots, \frac{N-1}{2}$ for odd p, we obtain

$$\begin{aligned}\chi[k, p] &\triangleq \chi(kT_c, \frac{p}{NT_c}) \\ &= e^{\mathrm{j}\pi \frac{p}{N}} \mathrm{sinc}\left(\pi \frac{p}{N}\right) \sum_{n=1}^{N} x_n x_{n-k}^* e^{-\mathrm{j}\pi(n-k)p/N}.\end{aligned} \tag{3}$$

Thus the **discrete-AF** can be defined as,

$$r[k, p] \triangleq \sum_{n=1}^{N} x_n x_{n-k}^* e^{-\mathrm{j}2\pi\frac{(n-k)p}{N}}. \tag{4}$$

We primarily be focus on designing the sequence $\{x_n\}_{n=1}^N$ so as to minimize the sidelobes of the discrete-AF in a certain region:

$$\begin{aligned}\mathcal{P}_1 : \ &\underset{\mathbf{x}}{\text{minimize}} \quad \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}} |r[k, p]|^2 \\ &\text{s.t. } \mathbf{x} \text{ is unimodular.}\end{aligned} \tag{5}$$

Theorem 1

The discrete-AF $r[k, p]$ can be reformulated as

$$r[k, p] = \mathbf{x}^{\mathbf{H}} \mathbf{D}_p \mathbf{J}_k \mathbf{x}, \tag{6}$$

where

$$\mathbf{D}_p = Diag\left(\left[e^{-j2\pi\frac{p}{N}}, \cdots, e^{-j2\pi\frac{(N-1)p}{N}}, e^{-j2\pi\frac{Np}{N}}\right]\right), \tag{7}$$

and

$$\mathbf{J}_k = \mathbf{J}_{-k}^{\mathbf{H}} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N-k} \\ \mathbf{I}_k & \mathbf{0} \end{bmatrix} \tag{8}$$

is the shift matrix that performs the shifting of the vector being multiplied by k lags.

With $\mathbf{A}_{k,p} = \mathbf{D}_p \mathbf{J}_k$ and

$$\begin{aligned}\mathbf{A}_{k,p}^r &\triangleq \frac{1}{2}(\mathbf{A}_{k,p} + \mathbf{A}_{k,p}^{\mathbf{H}}), \\ \mathbf{A}_{k,p}^i &\triangleq \frac{1}{2}(\mathbf{A}_{k,p} - \mathbf{A}_{k,p}^{\mathbf{H}})\end{aligned} \tag{9}$$

We arrive at the equivalent problem

$$\begin{aligned}\mathcal{P}_2 : \ &\underset{\mathbf{x}, \{\mathbf{u}_{k,p}^r\}, \{\mathbf{u}_{k,p}^i\}}{\text{minimize}} \quad \sum_{k,p} \left\{ \left\| (\tilde{\mathbf{A}}_{k,p}^r)^{1/2} \mathbf{x} - \sqrt{\zeta N} \mathbf{u}_{k,p}^r \right\|_2^2 \right. \\ &\qquad \left. + \left\| (\tilde{\mathbf{A}}_{k,p}^i)^{1/2} \mathbf{x} - \sqrt{\zeta N} \mathbf{u}_{k,p}^i \right\|_2^2 \right\} \\ &\text{s.t. } \mathbf{x} \text{ is unimodular,} \\ &\|\mathbf{u}_{k,p}^r\|_2 = \|\mathbf{u}_{k,p}^i\|_2 = 1 \text{ for all } k \in \mathcal{K}, p \in \mathcal{P},\end{aligned} \tag{10}$$

We follow a cyclic optimization approach to tackle $\mathcal{P}_2$ in an alternating manner over , $\{\mathbf{u}_{k,p}^r\}$ and $\{\mathbf{u}_{k,p}^i\}$. We have the closed-form solution for $\{\mathbf{u}_{k,p}^r\}$ and $\{\mathbf{u}_{k,p}^i\}$. Corresponding to each $k \in \mathcal{K}, p \in \mathcal{P}$:

$$\hat{\mathbf{u}}_{k,p}^{r(s)} = \frac{(\tilde{\mathbf{A}}_{k,p}^r)^{1/2} \mathbf{x}}{\|(\tilde{\mathbf{A}}_{l,p}^r)^{1/2} \mathbf{x}\|_2}, \tag{11}$$

$$\hat{\mathbf{u}}_{k,p}^{i(s)} = \frac{(\tilde{\mathbf{A}}_{k,p}^i)^{1/2} \mathbf{x}}{\|(\tilde{\mathbf{A}}_{k,p}^i)^{1/2} \mathbf{x}\|_2} : \tag{12}$$

$\mathcal{P}_2$ w.r.t $\mathbf{x}$ is equivalent to

$$\begin{aligned}&\max_{\mathbf{x}} \ \bar{\mathbf{x}}^{\mathbf{H}} \mathbf{D}_{\mathbf{x}} \bar{\mathbf{x}} \\ &\text{s.t. } |x_n| = 1, \quad n = 1, \cdots, N, \\ &\bar{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}.\end{aligned} \tag{13}$$

$\mathbf{D}_{\mathbf{x}}$, is a positive semidefinite matrix. The above problem is called **unimodular quadratic programming (UQP)** and the power-method-like iterations below leads to a monotonically decreasing objective value:

$$\mathbf{x}^{(s,t)} = \exp\left\{ \mathrm{j}\arg\left(\begin{bmatrix} \mathbf{I}_{N\times N} \\ \mathbf{0}_{1\times N} \end{bmatrix}^T \mathbf{D}_{\mathbf{x}} \bar{\mathbf{x}}^{(s,t-1)} \right) \right\} \tag{14}$$

Algorithm Radar code design for shaping the ambiguity function

Input: Index sets $\mathcal{K}$ and $\mathcal{P}$, $\mathbf{x}^{(0,0)}$, $\mathbf{u}_{k,p}^{r(0)}$, $\mathbf{u}_{k,p}^{i(0)}$ for $k \in \mathcal{K}$ and $p \in \mathcal{P}$.
Output: $\mathbf{x}$

for $t = 0 : \Gamma_1 - 1$ **do**
  for $s = 0 : \Gamma_2 - 1$ **do**
    Update $\mathbf{D}_x$
    $\mathbf{x}^{(t,s+1)} \leftarrow \exp\left\{ \mathrm{j}\arg\left(\begin{bmatrix} \mathbf{I}_{N\times N} \\ \mathbf{0}_{1\times N} \end{bmatrix}^T \mathbf{D}_{\mathbf{x}} \bar{\mathbf{x}}^{(t,s)} \right) \right\}$
    $\hat{\mathbf{u}}_{k,p}^{r(t+1)} \leftarrow \frac{(\tilde{\mathbf{A}}_{k,p}^r)^{1/2} \mathbf{x}^{(t,s)}}{\|(\tilde{\mathbf{A}}_{l,p}^r)^{1/2} \mathbf{x}^{(t,s)}\|_2}$,

    $\hat{\mathbf{u}}_{k,p}^{i(t+1)} \leftarrow \frac{(\tilde{\mathbf{A}}_{k,p}^i)^{1/2} \mathbf{x}^{(t,s)}}{\|(\tilde{\mathbf{A}}_{k,p}^i)^{1/2} \mathbf{x}^{(t,s)}\|_2}$.
return $\mathbf{x} \leftarrow \mathbf{x}^{(\Gamma_1, \Gamma_2)}$

Numerical Experiments

The region of interest is defined by the sets $\mathcal{K}$ and $\mathcal{P}$ as

$$\begin{aligned}&\mathcal{K} = \{5, 6, 7\} \quad \text{and} \\ &\mathcal{P} = \{-15, -14, -13, 11, 12, 13, 14\}.\end{aligned} \tag{15}$$

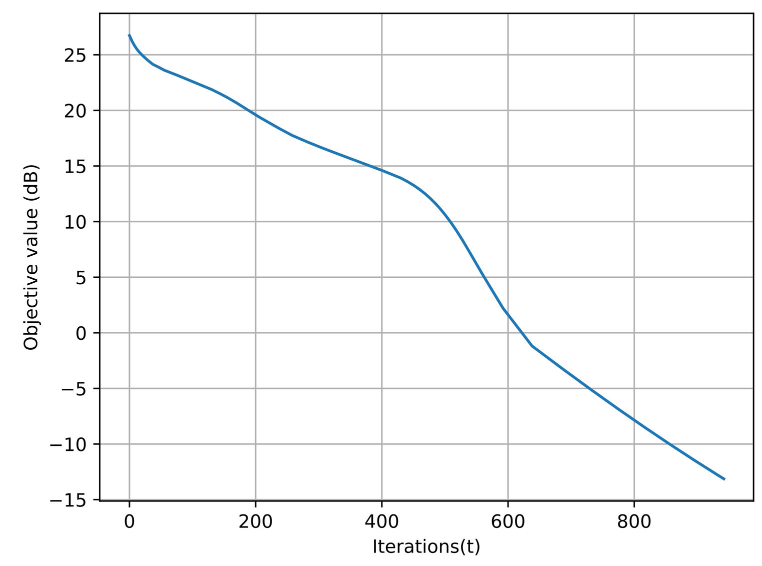
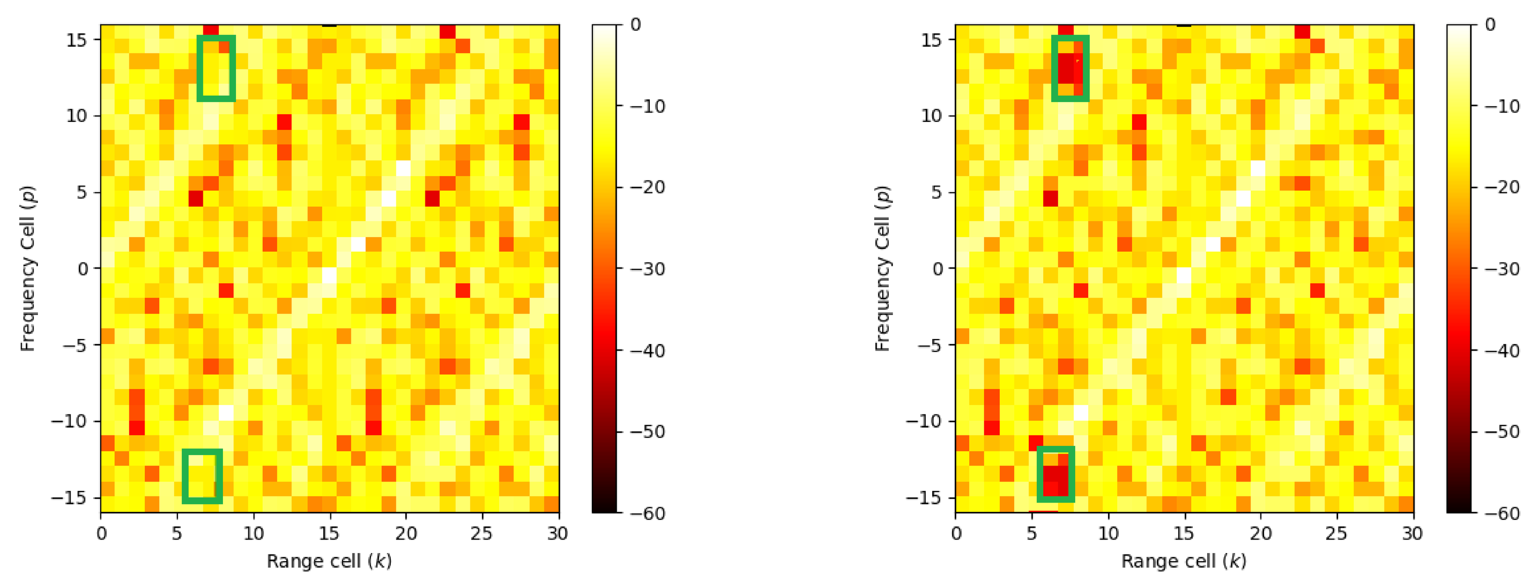


Figure. The objective value in (6) versus the iterations of Algorithm 1



(a) (b)

Figure. Ambiguity function, in dB, of (a) the initial random code and (b) the synthesized FMCW code with $N = 16$ and in green square the assumed regions of interest.

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