DESIGNING SIGNALS WITH GOOD CORRELATION AND DISTRIBUTION PROPERTIES

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1. Introduction

- Sequences with good correlation properties play an important role in various fields of signal processing applications including active sensing, spread spectrum communication systems, radar sensing, signal synchronization.
- Also, sequences with good correlation and distribution properties also have many important applications in biomedical signal and information processing [1].
- We propose an efficient computational framework for designing sequences with two key properties: ***(i)* an impulse like auto-correlation**, and ***(ii)* a probability distribution of sequence entries which are uniformly distributed.**

2. Application to Biomedical System Identification

- Unified Parkinson's Disease Rating Scale (UPDRS) is *(i)* very time consuming and is *(ii)* prone to human error [1].
- The new framework of eye tracking for quantifying the human smooth pursuit system (SPS) promises a solution to such difficulties.
- In an eye tracking system, the visual stimulus consists of a moving circle whose trajectory is the signal to be designed, and the eye's gaze direction is the output.
- In such scenarios, sequences with good correlation properties and a well defined distribution is required to identify the system with high degree of accuracy [2].

3. Problem Formulation

➢ Let $\boldsymbol{x} \in \mathbb{C}^N$ be a sequence whose aperiodic auto-correlations (r_k) are defined as

$$r_k \triangleq \sum_{n-1}^{N-k} x_n x_{n+k}^* = r_{-k}^* \; \forall\, 0 \le k \le N-1$$

The integrated sidelobe level (ISL) of the sequence $\boldsymbol{x}$ is defined as,

$$\text{ISL} \triangleq \sum_{k=1}^{N-1} |r_k|^2$$

Goal 1: To formulate algorithms for minimizing the ISL or ISL-related metrics over a set of sequences.

Goal 2: To achieve sequences with uniform distribution by partitioning the sequence entries into a number of range bins and populating each bin with (almost) same number of elements building a uniform histogram.

➢ A sequence $\boldsymbol{x} \in \mathbb{C}^N$ partitioned into P equi-spaced range bins denoted as $\{p_i\}_{i=1}^P$, can be called uniformly distributed if the number of elements in each bin, denoted as , $\mathcal{C}(p_i; \boldsymbol{x})$ follows:

$$\left|\mathcal{C}(p_i; \boldsymbol{x}) - \frac{N}{P}\right| \simeq 0, i \in \{1,2,\dots,P\}$$

where, $\mathcal{C}(E; \boldsymbol{x})$ is the counting function defined as the number of values $|x_n|$ $(1 \le n \le N)$ for which $\{x_n\} \in E$.

4. Construction Approach

➢ **Low Correlation:**

$$\min \|\boldsymbol{A}^H \boldsymbol{x} - \boldsymbol{v}\|_2^2$$

in the aperiodic case, $\widetilde{\boldsymbol{x}} = [\boldsymbol{x}\ \boldsymbol{0}_N]$,

$$\min \left\|\widetilde{\boldsymbol{A}}^H \widetilde{\boldsymbol{x}} - \widetilde{\boldsymbol{v}}\right\|_2^2$$

For a given $\widetilde{\boldsymbol{x}}$:

$$\widetilde{\boldsymbol{v}}^* = \frac{1}{\sqrt{2}} \exp(\arg(\widetilde{\boldsymbol{A}}^H \widetilde{\boldsymbol{x}}))$$

For a given $\widetilde{\boldsymbol{v}}$:

$$\widetilde{\boldsymbol{x}}^* = \widetilde{\boldsymbol{A}} \widetilde{\boldsymbol{v}}$$

➢ **Uniform Distribution:**

We first sort the sequence entries in an ascending order to form

$$\boldsymbol{h} = \mathcal{S}_a(\widetilde{\boldsymbol{x}})$$

Then partition $\boldsymbol{h}$ into P equisized range bins, where $1 \le P \le N$.

Algorithm 1:

```
Data: h, N, P
Result: ĥ ≜ {ĥ_n}_{n=1}^N
initialize n = 1;
maxnum ← floor(N/P);
while n ≤ N do
    bin_index ← ceil(n/maxnum);
    if h_n < lower_edge(bin_index) then
        ĥ_n ← lower_edge(bin_index);
    else if h_n > upper_edge(bin_index) then
        ĥ_n ← upper_edge(bin_index);
    else
        ĥ_n ← h_n;
    end
    n ← n + 1;
end
```

5. Final Algorithm

Input parameters: N, P.
Step 0: Initialize $\boldsymbol{x}$ using a randomly generated sequence.
Step 1: Compute $\widetilde{\boldsymbol{v}}^* = \frac{1}{\sqrt{2}} \exp(\arg(\widetilde{\boldsymbol{A}}^H \widetilde{\boldsymbol{x}}))$.
Step 2: Compute $\widetilde{\boldsymbol{x}}^* = \widetilde{\boldsymbol{A}} \widetilde{\boldsymbol{v}}$.
Step 3: Compute $\boldsymbol{h} = \mathcal{S}_a(\widetilde{\boldsymbol{x}}^*)$ and preserve the index of each elements of original sequence $\widetilde{\boldsymbol{x}}^*$ in $\mathcal{I}_a(\widetilde{\boldsymbol{x}}^*)$.
Step 4: Partition $\boldsymbol{h}$ into P bins of equal length and compute the edges of each bin.
Step 5: Modify the elements of $\boldsymbol{h}$ using Algorithm 1.
Step 6: Compute $\widetilde{\boldsymbol{x}}$ by restoring the index order previously stored in step 3.
Step 7: Let $d = \|\boldsymbol{A}^H \boldsymbol{x} - \boldsymbol{v}\|$. Repeat steps 1–6 until $\left|d^{(s)} - d^{(s-1)}\right| \le 10^{-6}$ where s denotes the iteration number.

7. Results

We use the proposed approach to design sequences of length *(i)* $N = 10^3$ with number of partitions $P = 20$, and *(ii)* $N = 10^4$ with $P = 250$.

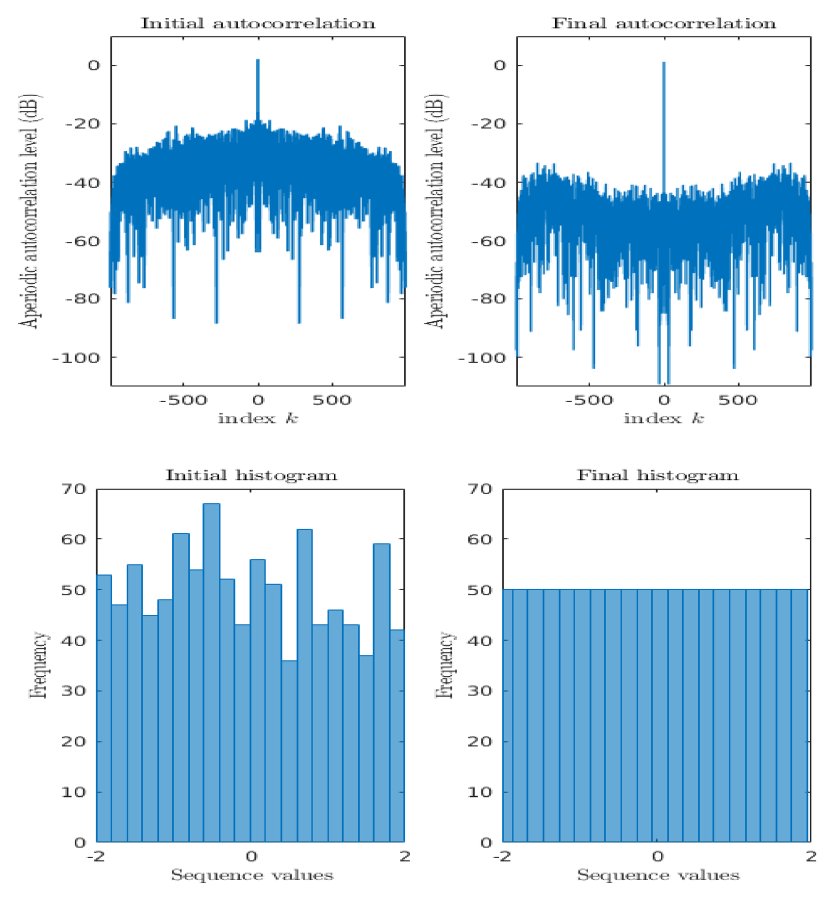

Fig.1 The initial and final normalized aperiodic auto-correlation, and histogram of constructed sequence of length $N = 10^3$ and $P = 20$.

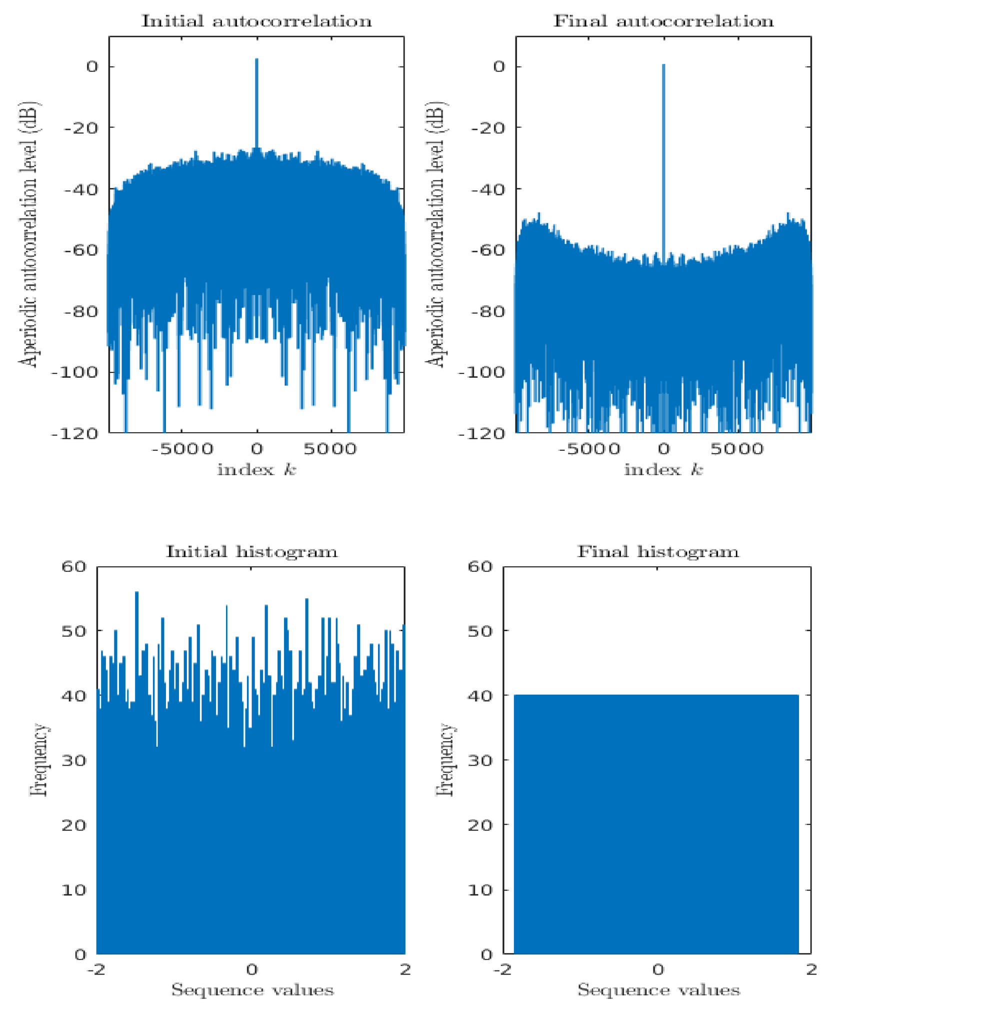

Fig. 2 The initial and final normalized aperiodic auto-correlation, and histogram of constructed sequence of length $N = 10^4$ and $P = 250$.

9. Conclusion

We have presented a new framework to design sequences with good correlation and distribution properties based on the CAN computational framework. The proposed method is computationally efficient and can design very long sequences (of lengths up to N ~ 10^6 and even more) in relatively short time frames.

10. References

[1] D. Jansson, Mathematical modeling of the human smooth pursuit system, Ph.D. thesis, Uppsala University, Division of Systems and Control, Automatic control, 2014.

[2] P. Stoica and R.L. Moses, Introduction to Spectral Analysis, Prentice Hall, 1997.