Joint Optimization of Waveform Covariance Matrix and Antenna Selection for MIMO Radar

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MIMO and active sensing

It's been over 10 years since the benefits of MIMO has been recognized:

- Virtual spatial channels, an adaptive degree of freedom.
- Broadening of the transmitter beam pattern.
- Rapid detection and mitigation of strong clutter discretes.
- Jointly optimize both the transmit and receive DoF.
Jointly exploit Tx-Rx DoF

**Couple of ways...**

- Maximize SICR by jointly designing the probing signal and the receive filter coefficients.
- Control distribution of transmit power by approximating a desired beampattern by optimizing the transmit covariance matrix.
Jointly exploit Tx-Rx DoF

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- Control distribution of transmit power by approximating a desired beampattern by optimizing the transmit covariance matrix.

Why transmit covariance?

- Extra degrees of freedom.
- Acts as an oracle for waveform design problem.
- Need low cross-correlation sidelobe? No problem.
The traditional case

Uniform linear array (ULA)
What we are up to

The spatial diversity

- Antenna position and/or alignment introduces additional degrees of freedom.
- Smart antenna position designing can save a lot of resources[1].

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018
Let’s call it *non-uniform linear array (NULA)*

When do we require it?
- Adaptive beamforming for autonomous vehicle.
- Aerial beamforming using drones.
- Localization applications.
The goal is to...

- Jointly design
  - the covariance matrix
  - antenna selection vector
- Match a desired beam pattern
- Minimize cross-correlation sidelobe.
Preliminaries

Antenna selection vector:

\[ \mathbf{p} = [p_1, p_2, \cdots, p_M]^T, \quad p_m \in \{0, 1\} \]

Steering vector:

\[ \mathbf{a}(\theta) = [1, e^{j \frac{2\pi}{\lambda} d \sin \theta}, \cdots, e^{j \frac{2\pi}{\lambda} (M-1)d \sin \theta}]^T \]

Space-time transmit waveform:

\[ \mathbf{s}(l) = [s_1(l), s_2(l), \cdots, s_M(l)]^T \]

The baseband waveform at azimuth location \( \theta \):

ULA: \[ x(l) = \mathbf{a}(\theta)^H \mathbf{s}(l) \]

NULA: \[ x(l) = (\mathbf{p} \odot \mathbf{a}(\theta))^H \mathbf{s}(l), \quad l \in \{1, \cdots, L\}. \]
The power produced by the waveform at $\theta$

$$P(\theta) = \mathbb{E}\{|x(l)|^2\}$$

$$= (p \odot a(\theta))^H \mathbb{E}\{s(l)s^H(l)\}(p \odot a(\theta))$$

$$= p^T \text{Re} \left\{ R \odot (a(\theta)a^H(\theta))^* \right\} p,$$

where

$$R = \mathbb{E}\left\{s(l)s^H(l)\right\},$$
The power produced by the waveform at $\theta$

$$P(\theta) = \mathbb{E}\{|x(l)|^2\} = (p \odot a(\theta))^H \mathbb{E}\{s(l)s^H(l)\}(p \odot a(\theta)) = p^T \text{Re} \left\{ R \odot \left(a(\theta)a^H(\theta)\right)^* \right\} p,$$

where

$$R = \mathbb{E}\left\{ s(l)s^H(l) \right\},$$

and the cross-correlation terms between $\theta$ and $\bar{\theta}$

$$\bar{P}(\theta, \bar{\theta}) \triangleq p^T \text{Re} \left\{ R \odot \left(a(\theta)a^H(\bar{\theta})\right)^* \right\} p.$$
The desired beampattern $d(\theta)$

Assume some partial information regarding the target positions $\{\hat{\theta}_k\}_{k=1}^{\hat{K}}$ are known.

$$d(\theta) = \begin{cases} 
1, & \theta \in [\hat{\theta}_k - \frac{\triangle}{2}, \hat{\theta}_k + \frac{\triangle}{2}], \quad k \in \{1, \cdots, \hat{K}\}, \\
0, & \text{otherwise},
\end{cases}$$

$$\hat{\theta} = [-50^\circ, 0^\circ, 50^\circ]$$

$$\triangle = 20^\circ$$
The objective function

\[ J(p, R, \alpha) = \frac{1}{K} \sum_{k=1}^{K} w_k \left| p^T \text{Re} \left\{ R \odot \left( a(\theta_k) a^H(\theta_k) \right)^* \right\} p - \alpha d(\theta_k) \right|^2 \]

beampattern matching term

\[ + \frac{2 \omega_c}{\hat{K}(\hat{K} - 1)} \sum_{p=1}^{\hat{K}-1} \sum_{q=p+1}^{\hat{K}} \left| p^T \text{Re} \left\{ R \odot \left( a(\hat{\theta}_p) a^H(\hat{\theta}_q) \right)^* \right\} p \right|^2 \]

cross-correlation term
Problem formulation (contd.)

The optimization formulation

\[
\begin{align*}
\min_{R, p, \alpha} & \quad J(R, p, \alpha) \\
\text{s.t.} & \quad R \succeq 0, \\
& \quad R_{mm} = \frac{c}{M}, \quad \text{for } m = 1, \ldots, M, \\
& \quad \|p\|_1 = N, \\
& \quad p_m = \{0, 1\}, \quad \text{for } m = 1, \ldots, M, \\
& \quad \alpha > 0.
\end{align*}
\]
Optimization of $\mathbf{R}$ and $\alpha$

\[
\left( \mathbf{R}^{(t)}, \alpha^{(t)} \right) = \arg\min_{\mathbf{R},\alpha} J(p^{(t-1)}, \mathbf{R}, \alpha)
\]

s.t. $\mathbf{R} \succeq 0$, 

\[
R_{mm} = \frac{c}{M}, \text{ for } m = 1, \cdots, M,
\]

$\alpha > 0$.

- Can be formulated as a constrained convex quadratic program.
- Any convex optimization toolbox e.g. CVX for Matlab, CVXPY, CVXOPT for Python can be used.
Optimization of $p$

$$p^{(t+1)} = \arg \min_p J(p, R^{(t)}, \alpha^{(t)}),$$

s.t. $\|p\|_1 = N,$

$$p \in \{0, 1\}^M.$$ 

- Binary optimization problem (NP hard).
Optimization of $p$

$$p^{(t+1)} = \arg\min_p J(p, R^{(t)}, \alpha^{(t)}),$$

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- Binary optimization problem (NP hard).
- Does convex relaxation work?
  - Relax $p$ into $[0, 1]$, optimize for $p$, then map it back to $\{0, 1\}^M$ using hard thresholding.
  - No exact solution.
  - The solution is not always consistent.
  - Search on real number makes the search space too big to handle.
Optimization of $p$

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  - No exact solution.
  - The solution is not always consistent.
  - Search on real number makes the search space too big to handle.
- Need *sophisticated tool* to properly handle it.
We propose a tool inspired from *dynamic programming* and *evolutionary algorithm*.
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For a given \((\mathbf{R}, \alpha)\), the solution of \(J(p)\): \(p\) is a binary vector of length \(M\) with \(N\) non-zero elements.
Optimization of $p$ (contd.)

- We propose a tool inspired from *dynamic programming* and *evolutionary algorithm*.
- For a given $(\mathbf{R}, \alpha)$, the solution of $J(p)$: $p$ is a binary vector of length $M$ with $N$ non-zero elements.
- In other words, our search space is a subset of vertices of a hypercube in an $M$-dimensional space.
We propose a tool inspired from *dynamic programming* and *evolutionary algorithm*.

For a given \((\mathbf{R}, \alpha)\), the solution of \(J(\mathbf{p})\): \(\mathbf{p}\) is a binary vector of length \(M\) with \(N\) non-zero elements.

In other words, our search space is a subset of vertices of a hypercube in an \(M\)-dimensional space.

Given the solution \(\mathbf{p}^{(k)}\) (*parent solution*), a new set of *candidate solutions* \(\mathbf{p}_{CS}^{(k+1)}\) is generated as:

\[
\mathbf{p}_{CS}^{(k+1)} = \left\{ \mathbf{p} \mid H \left( \mathbf{p}, \mathbf{p}^{(k)} \right) = 1, \| \mathbf{p} \|_1 < \| \mathbf{p}^{(k)} \|_1 \right\}.
\]
Optimization of $p$ (contd.)

- The cardinality of the new candidate solution is upper bounded by $|p(k+1)_{CS}| \leq \|p(k)\|_1$.

Select and propagate the best candidate solution: $p(k)=\arg\min_{p \in p(k)_{CS}} J(p)$.

- **red** vertex $\Rightarrow$ the parent solution,
- **yellow** vertices $\Rightarrow$ the candidate solutions $p_{CS}$,
- **blue** vertex $\Rightarrow$ the selected solution for the next iteration.
Optimization of $p$ (contd.)

- The cardinality of the new candidate solution is upper bounded by $\left| p_{CS}^{(k+1)} \right| \leq \| p^{(k)} \|_1$.
- Select and propagate the best candidate solution: $p^{(k)} = \arg \min_{p \in p_{CS}^{(k)}} J(p)$. 

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The algorithm

**Table:** The Proposed Joint Optimization Method

<table>
<thead>
<tr>
<th>Step 0:</th>
<th>Initialize the antenna position vector $p^{(0)} = 1_M$, the complex covariance matrix $R^{(0)} \in \mathbb{C}^{N \times N}$, and the scaling factor $\alpha^{(0)} \in \mathbb{R}_+$, and the outer loop index $t = 1$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1:</td>
<td>Solve the convex program for $R, \alpha$ and obtain $(R^{(t)}, \alpha^{(t)})$.</td>
</tr>
<tr>
<td>Step 2:</td>
<td>Employ the proposed binary optimization approach for $p$ to obtain the vector $p^{(t+1)}$.</td>
</tr>
<tr>
<td>Step 3:</td>
<td>Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $H(p^{(t)}, p^{(t-1)}) = 0$.</td>
</tr>
</tbody>
</table>
Numerical examples

Experimental setup I:

\[ M = 15, \, N = 10, \, \hat{\theta} = \{-50^\circ, 0^\circ, 50^\circ\}, \, \triangle = 20^\circ \]

\[ \text{Figure: The transmit beampattern design} \]

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018
Numerical examples

Experimental setup I:
\[ M = 15, N = 10, \hat{\theta} = \{-50^\circ, 0^\circ, 50^\circ\}, \triangle = 20^\circ \]

Figure: Normalized crosscorrelation coefficients

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Numerical examples

Experimental setup I:

\[ M = 15, \; N = 10, \; \hat{\theta} = \{-50^\circ, 0^\circ, 50^\circ\}, \; \triangle = 20^\circ \]

**Figure:** Final antenna positions

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018
Experimental setup II:

\[ M = 15, N = 10, \hat{\theta} = \{0^\circ\}, \triangle = 60^\circ \]

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018
Experimental setup III:

\[ M = 20, \ N = 15, \ \hat{\theta} = \{-60^\circ, -30^\circ, 0^\circ, 30^\circ, 60^\circ\}, \ \Delta = 10^\circ \]
Computational cost

We consider $M = 4$ and $N = 3$ as initialization, and then linearly scale $M$ and $N$ by the factor of $\beta \in \{1, 2, 3, 4\}$.

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018
Summary

- We jointly design the probing signal covariance matrix as well as the antenna positions to approximate a given beampattern while minimizing the cross-correlation sidelobe.
- We propose a binary optimization framework based on dynamic programming which is realizable in polynomial time.
- The algorithm is highly parallelizable and scalable.
Thank you and Questions?