Joint Optimization of Waveform Covariance Matrix and Antenna Selection for MIMO Radar

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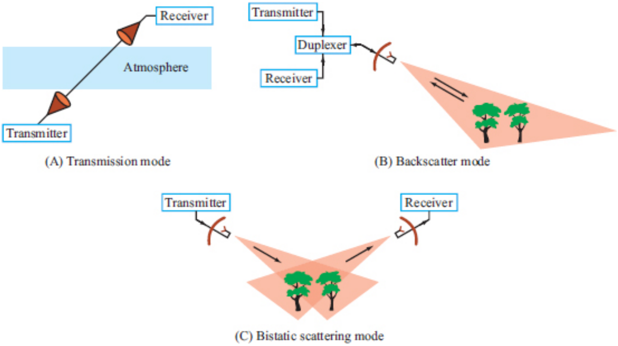
MIMO and active sensing

Its been over 10 years since the benefits of MIMO has been recognized

- Virtual spatial channels, an adaptive degree of freedom.
- Broadening of the transmitter beam pattern.
- Rapid detection and mitigation of strong clutter discretes.
- Jointly optimize both the transmit and receive DoF.



Jointly exploit Tx-Rx DoF

Couple of ways...

- Maximize SICR by jointly designing the probing signal and the receive filter coefficients.
- Control distribution of transmit power by approximating a desired beampattern by optimizing the transmit covariance matrix.

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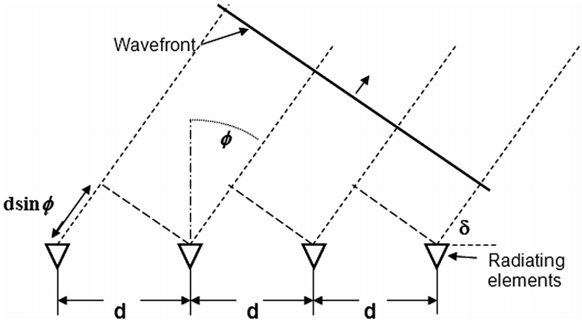
Why transmit covariance?

- Extra degrees of freedom.
- Acts as an oracle for waveform design problem.
- Need low cross-correlation sidelobe? No problem.

The traditional case

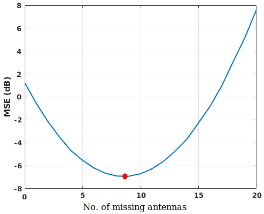
Uniform linear array (ULA)



What we are up to

The spatial diversity

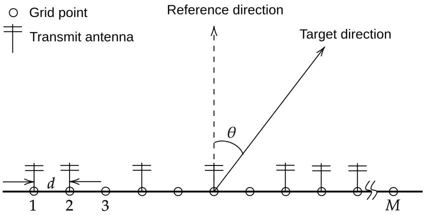
- Antenna position and/or alignment introduces additional degrees of freedom.
- Smart antenna position designing can save a lot of resources[1].



[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018

NULA

Let's call it *non-uniform linear array (NULA)*



When do we require it?

- Adaptive beamforming for autonomous vehicle.
- Aerial beamforming using drones.
- Localization applications.

Objective

The goal is to...

- Jointly design
  - the covariance matrix
  - antenna selection vector
- Match a desired beam pattern
- Minimize cross-correlation sidelobe.

Preliminaries

Antenna selection vector:

$$\boldsymbol{p} = [p_1, p_2, \cdots, p_M]^T, p_m \in \{0, 1\}$$

Steering vector:

$$\boldsymbol{a}(\theta) = [1, e^{j\frac{2\pi}{\lambda} d \sin\theta}, \cdots, e^{j\frac{2\pi}{\lambda}(M-1) d \sin\theta}]^T$$

Space-time transmit waveform:

$$\boldsymbol{s}(l) = [s_1(l), s_2(l), \cdots, s_M(l)]^T$$

The baseband waveform at azimuth location θ:

$$\begin{aligned} \text{ULA}: \ x(l) = & \quad \boldsymbol{a}(\theta)^H \boldsymbol{s}(l) \\ \text{NULA}: \ x(l) = & \quad (\boldsymbol{p} \odot \boldsymbol{a}(\theta))^H \boldsymbol{s}(l), \qquad l \in \{1, \cdots, L\}. \end{aligned}$$

Preliminaries (contd.)

The power produced by the waveform at θ

$$\begin{aligned} P(\theta) &= \mathbb{E}\{|x(l)|^2\} \\ &= (\boldsymbol{p} \odot \boldsymbol{a}(\theta))^H \mathbb{E}\{\boldsymbol{s}(l)\boldsymbol{s}^H(l)\}(\boldsymbol{p} \odot \boldsymbol{a}(\theta)) \\ &= \boldsymbol{p}^T \operatorname{Re}\left\{\boldsymbol{R} \odot \left(\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)\right)^*\right\}\boldsymbol{p}, \end{aligned}$$

where

$$\boldsymbol{R} = \mathbb{E}\left\{\boldsymbol{s}(l)\boldsymbol{s}^H(l)\right\},$$

Preliminaries (contd.)

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where

$$\boldsymbol{R} = \mathbb{E}\left\{\boldsymbol{s}(l)\boldsymbol{s}^H(l)\right\},$$

and the cross-correlation terms between θ and $\bar{\theta}$

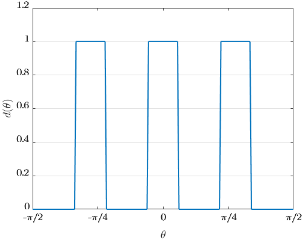
$$\bar{P}(\theta, \bar{\theta}) \triangleq \boldsymbol{p}^T \operatorname{Re}\left\{\boldsymbol{R} \odot \left(\boldsymbol{a}(\theta)\boldsymbol{a}^H(\bar{\theta})\right)^*\right\} \boldsymbol{p}.$$

Problem formulation

The desired beampattern $d(\theta)$

Assume some partial information regarding the target positions $\{\hat{\theta}_k\}_{k=1}^{\hat{K}}$ are known.

$$d(\theta) = \begin{cases} 1, & \theta \in [\hat{\theta}_k - \frac{\triangle}{2}, \hat{\theta}_k + \frac{\triangle}{2}], \quad k \in \{1, \cdots, \hat{K}\}, \\ 0, & \text{otherwise}, \end{cases}$$



$\hat{\theta} = [-50^\circ, 0^\circ, 50^\circ]$

$\triangle = 20^\circ$

The objective function

$$J(\boldsymbol{p}, \boldsymbol{R}, \alpha) = \underbrace{\frac{1}{K}\sum_{k=1}^{K} w_k \left| \boldsymbol{p}^T \operatorname{Re}\left\{ \boldsymbol{R} \odot \left(\boldsymbol{a}(\theta_k)\boldsymbol{a}^H(\theta_k) \right)^* \right\} \boldsymbol{p} - \alpha d(\theta_k) \right|^2}_{\text{beampattern matching term}}$$

$$+ \underbrace{\frac{2\omega_c}{\hat{K}(\hat{K}-1)} \sum_{p=1}^{\hat{K}-1} \sum_{q=p+1}^{\hat{K}} \left| \boldsymbol{p}^T \operatorname{Re}\left\{ \boldsymbol{R} \odot \left(\boldsymbol{a}(\hat{\theta}_p)\boldsymbol{a}^H(\hat{\theta}_q) \right)^* \right\} \boldsymbol{p} \right|^2}_{\text{cross-correlation term}}$$

Problem formulation (contd.)

The optimization formulation

$$
\begin{aligned}
\min_{\boldsymbol{R},\boldsymbol{p},\alpha} \quad & J(\boldsymbol{R},\boldsymbol{p},\alpha) \\
\text{s.t.} \quad & \boldsymbol{R} \succeq \mathbf{0}, \\
& R_{mm} = \frac{c}{M}, \quad \text{for } m = 1,\cdots,M, \\
& \|\boldsymbol{p}\|_1 = N, \\
& p_m = \{0,1\}, \quad \text{for } m = 1,\cdots,M, \\
& \alpha > 0.
\end{aligned}
$$

Optimization of $\boldsymbol{R}$ and α

$$\left(\boldsymbol{R}^{(t)}, \alpha^{(t)}\right) = \arg\min_{\boldsymbol{R},\alpha} J(\boldsymbol{p}^{(t-1)}, \boldsymbol{R}, \alpha)$$

$$\text{s.t. } \boldsymbol{R} \succeq \boldsymbol{0},$$

$$R_{mm} = \frac{c}{M}, \text{ for } m = 1, \cdots, M,$$

$$\alpha > 0.$$

- Can be formulated as a constrained convex quadratic program.
- Any convex optimization toolbox e.g. `CVX` for Matlab, `CVXPY`, `CVXOPT` for Python can be used.

Optimization of $\boldsymbol{p}$

$$
\begin{aligned}
\boldsymbol{p}^{(t+1)} = \arg\min_{\boldsymbol{p}} \; & J(\boldsymbol{p}, \boldsymbol{R}^{(t)}, \alpha^{(t)}), \\
\text{s.t. } & \|\boldsymbol{p}\|_1 = N, \\
& \boldsymbol{p} \in \{0,1\}^M.
\end{aligned}
$$

- Binary optimization problem (NP hard).

Optimization of $\boldsymbol{p}$

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- Binary optimization problem (NP hard).
- Does convex relaxation work?
  - Relax p into $[0,1]$, optimize for $\boldsymbol{p}$, then map it back to $\{0,1\}^M$ using hard thresholding.
  - No exact solution.
  - The solution is not always consistent.
  - Search on real number makes the search space too big to handle.

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  - No exact solution.
  - The solution is not always consistent.
  - Search on real number makes the search space too big to handle.
- Need *sophisticated tool* to properly handle it.

Optimization of $\boldsymbol{p}$ (contd.)

- We propose a tool inspired from *dynamic programming* and *evolutionary algorithm*.

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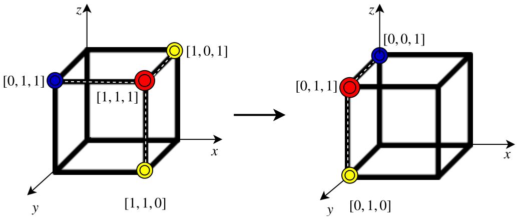
- For a given $(\boldsymbol{R}, \alpha)$, the solution of $J(\boldsymbol{p})$: $\boldsymbol{p}$ is a binary vector of length M with N non-zero elements.
- In other words, our search space is a subset of vertices of a hypercube in an M-dimensional space.
- Given the solution $\boldsymbol{p}^{(k)}$ (*parent solution*), a new set of *candidate solutions* $\boldsymbol{p}_{\text{CS}}^{(k+1)}$ is generated as:

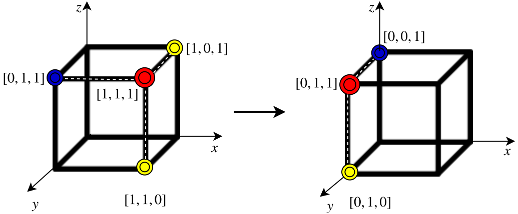
$$\boldsymbol{p}_{\text{CS}}^{(k+1)} = \left\{ \boldsymbol{p} \mid H\left(\boldsymbol{p}, \boldsymbol{p}^{(k)}\right) = 1,\ \|\boldsymbol{p}\|_1 < \|\boldsymbol{p}^{(k)}\|_1 \right\}.$$

Optimization of $\boldsymbol{p}$ (contd.)



red vertex $\Rightarrow$ the parent solution,
yellow vertices $\Rightarrow$ the candidate solutions $\boldsymbol{p}_{\text{CS}}$,
blue vertex $\Rightarrow$ the selected solution for the next iteration.

Optimization of $\boldsymbol{p}$ (contd.)



red vertex $\Rightarrow$ the parent solution,
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- The cardinality of the new candidate solution is upper bounded by $\left|\boldsymbol{p}_{\text{CS}}^{(k+1)}\right| \leq \|\boldsymbol{p}^{(k)}\|_1$.
- Select and propagate the best candidate solution: $\boldsymbol{p}^{(k)} = \arg\min_{\boldsymbol{p}\in\boldsymbol{p}_{\text{CS}}^{(k)}} J(\boldsymbol{p})$.

The algorithm

Table: The Proposed Joint Optimization Method

Step 0: Initialize the antenna position vector $\boldsymbol{p}^{(0)} = \mathbf{1}_M$, the complex covariance matrix $\boldsymbol{R}^{(0)} \in \mathbb{C}^{N\times N}$, and the scaling factor $\alpha^{(0)} \in \mathbb{R}_+$, and the outer loop index $t = 1$.

Step 1: Solve the convex program for $\boldsymbol{R}, \alpha$ and obtain $\left(\boldsymbol{R}^{(t)}, \alpha^{(t)}\right)$.

Step 2: Employ the proposed binary optimization approach for $\boldsymbol{p}$ to obtain the vector $\boldsymbol{p}^{(t+1)}$.

Step 3: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $H\left(\boldsymbol{p}^{(t)}, \boldsymbol{p}^{(t-1)}\right) = 0$.

Numerical examples

Experimental setup I:

$M = 15, N = 10, \hat{\theta} = \{-50^\circ, 0^\circ, 50^\circ\}, \triangle = 20^\circ$

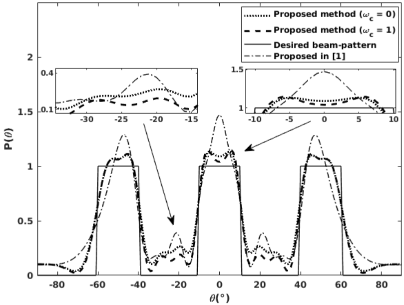

Figure: The transmit beampattern design

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018

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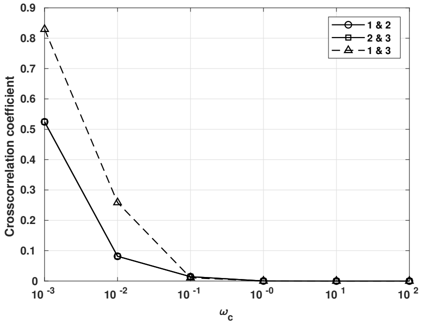

Figure: Normalized crosscorrelation coefficients

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018

Numerical examples

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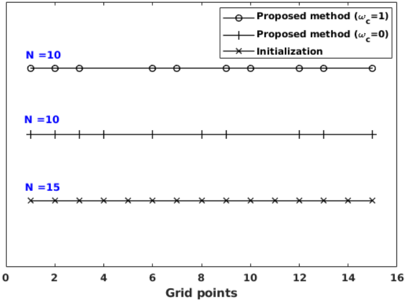


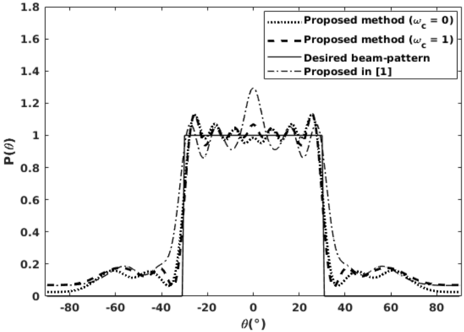
Figure: Final antenna positions

[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018

Numerical examples (contd.)

Experimental setup II:

$M = 15, N = 10, \hat{\theta} = \{0°\}, \triangle = 60°$

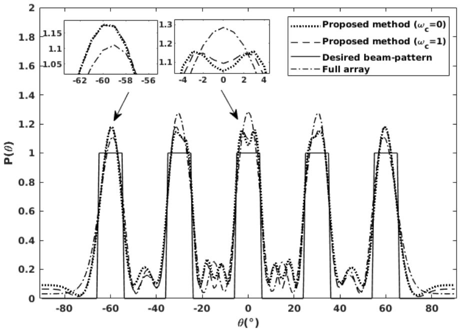


[1] Z. Cheng et al. Joint optimization of covariance matrix and antenna position for MIMO radar transmit beampattern matching design, 2018

Numerical examples (contd.)

Experimental setup III:

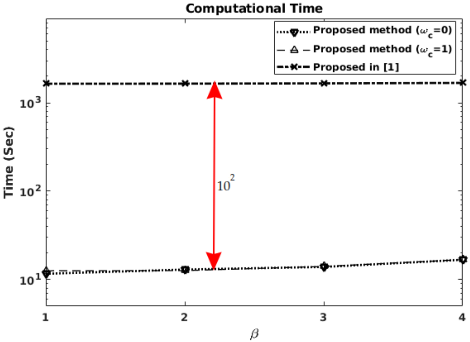
$M = 20, N = 15, \hat{\theta} = \{-60^\circ, -30^\circ, 0^\circ, 30^\circ, 60^\circ\}, \triangle = 10^\circ$



Numerical examples (contd.)

Computational cost

We consider $M = 4$ and $N = 3$ as initialization, and then linearly scale M and N by the factor of $\beta \in \{1, 2, 3, 4\}$.



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Summary

- We jointly design the probing signal covariance matrix as well as the antenna positions to approximate a given beampattern while minimizing the cross-correlation sidelobe.
- We propose a binary optimization framework based on dynamic programming which is realizable in polynomial time.
- The algorithm is highly parallelizable and scalable.

Thank you
and
Questions?