Deep Radar Waveform Design for Efficient Automotive Radar Sensing

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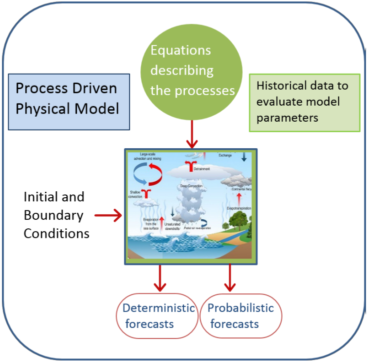
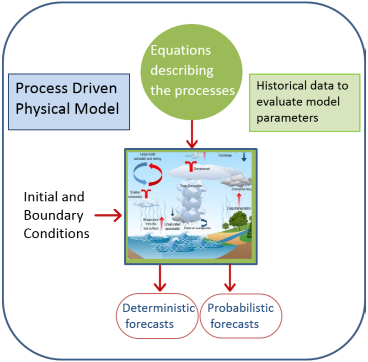
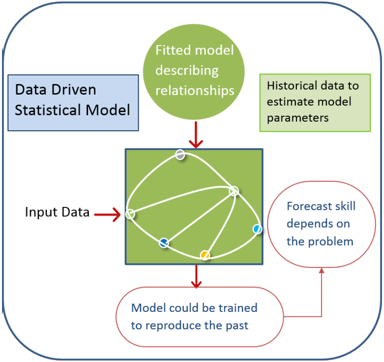


Model-Based and Data-Driven Approaches





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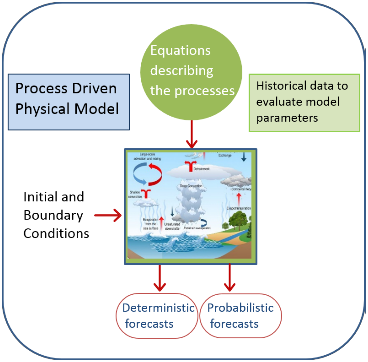

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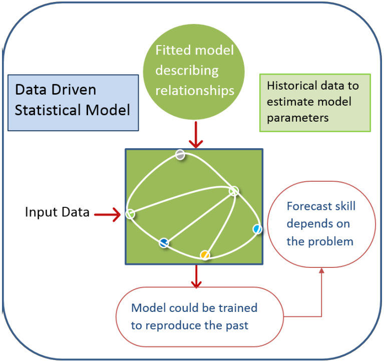
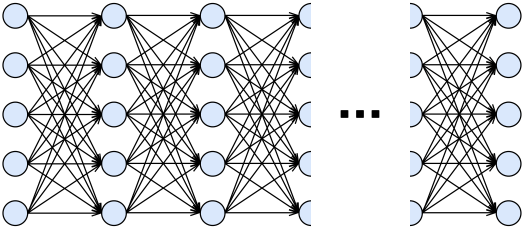


DNN and DUN

Deep Neural Network (DNN)



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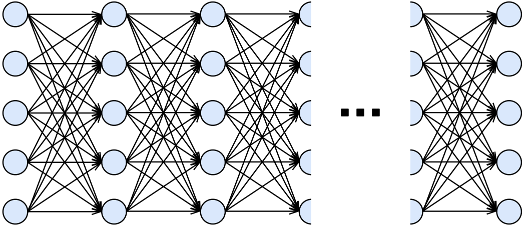
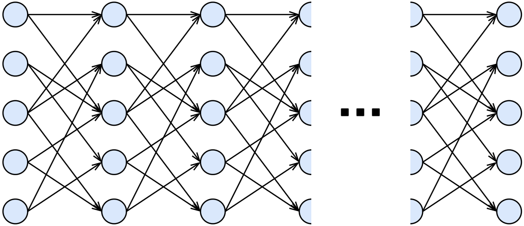


Deep Unfolding Network (DUN)



Signal Model

Sequence:

$$\boldsymbol{s} = [s_1 \ s_2 \ \cdots \ s_N]^T \in \mathbb{C}^N$$

Output:

$$\boldsymbol{y} = \boldsymbol{A}^H \boldsymbol{\alpha} + \boldsymbol{\epsilon}$$

where

$$\boldsymbol{A}^H = \begin{bmatrix} s_1 & 0 & \cdots & 0 & s_N & s_{N-1} & \cdots & s_2 \\ s_2 & s_1 & & \vdots & 0 & s_N & & \vdots \\ \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \ddots & s_N \\ s_N & s_{N-1} & \cdots & s_1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\boldsymbol{\alpha} = [\alpha_0 \ \alpha_1 \ \cdots \ \alpha_{N-1} \ \alpha_{-N+1} \ \cdots \ \alpha_{-1}]^T \in \mathbb{C}^{2N-1}.$$

Problem Formulation

In the pulse compression stage, using an MF:

$$f(\boldsymbol{s}) \triangleq \frac{|\boldsymbol{s}^H \boldsymbol{y}|^2}{\sum_{k \neq 0} |\boldsymbol{s}^H \boldsymbol{J}_k \boldsymbol{y}|^2} = \frac{\boldsymbol{s}^H \boldsymbol{A} \boldsymbol{s}}{\boldsymbol{s}^H \boldsymbol{B} \boldsymbol{s}} \triangleq \frac{n(\boldsymbol{s})}{d(\boldsymbol{s})}$$

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The optimization problem:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \frac{\boldsymbol{s}^H \boldsymbol{A} \boldsymbol{s}}{\boldsymbol{s}^H \boldsymbol{B} \boldsymbol{s}} \\ \text{s.t.} \quad & |s_k| = 1, \quad k \in \{1, \ldots, N\} \end{aligned}$$

Problem Formulation (Contd.)

Unimodular quadratic program (UQP):

$$\max_{\boldsymbol{s}} \quad \boldsymbol{s}^H \boldsymbol{\chi} \boldsymbol{s}$$
$$\text{s.t.} \quad |s_k| = 1, \quad k \in \{1, \ldots, N\}.$$

[1] M. Soltanalian et al. "Joint design of the receive filter and transmit sequence for active sensing", IEEE Signal Processing Letters, vol. 20, no. 5, pp. 423–426, May 2013.

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Power method like iterations (PMLI)[1]:

$$\min_{\boldsymbol{s}^{(n+1)}} \left\| \boldsymbol{s}^{(n+1)} - \boldsymbol{\chi} \boldsymbol{s}^{(n)} \right\|_2,$$
$$\text{s.t.} \quad \left| s_k^{(n+1)} \right| = 1, \, \forall \, k$$

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Analytical solution:

$$\boldsymbol{s}^{(n+1)} = e^{j \arg(\boldsymbol{\chi} \boldsymbol{s}^{(n)})}$$

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Fully Connected DNN

Let,

$$\tilde{g}_{\phi_i}(\boldsymbol{z}) = a(\boldsymbol{u})$$

where $\boldsymbol{u} = \boldsymbol{W}_i\boldsymbol{z}$, $\phi_i = \{\boldsymbol{W}_i\}$, and $a(\cdot)$ denotes a non-linear activation function

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Given an input $\boldsymbol{x}_0$, the dynamics of a fully connected DNN with L layers:

$$\boldsymbol{x}_L = \mathcal{F}(\boldsymbol{x}_0; \boldsymbol{\Upsilon}) = \tilde{g}_{\phi_{L-1}} \circ \tilde{g}_{\phi_{L-2}} \circ \cdots \circ \tilde{g}_{\phi_0}(\boldsymbol{x}_0),$$
$$\boldsymbol{\Upsilon} = \{\phi_i\}_{i=0}^{L-1}$$

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PMLI: $\mathcal{S}(\boldsymbol{x}) \triangleq e^{j \arg(\boldsymbol{x})}$

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PMLI: $\mathcal{S}(\boldsymbol{x}) \triangleq e^{j\arg(\boldsymbol{x})}$

PMLI are perfect candidates for unfolding onto a DUN

Let,

$$g_{\phi_i}(\boldsymbol{z}) = \mathcal{S}(\boldsymbol{u})$$

where $\boldsymbol{u} = \boldsymbol{\chi}_i \boldsymbol{z}$ and $\phi_i = \{\boldsymbol{\chi}_i\}$

Deep Evolutionary Cognitive Radar (DECoR)

Let,

$$g_{\phi_i}(\boldsymbol{z}) = \mathcal{S}(\boldsymbol{u})$$

where $\boldsymbol{u} = \boldsymbol{\chi}_i \boldsymbol{z}$ and $\phi_i = \{\boldsymbol{\chi}_i\}$
The dynamics of DECoR for L layers:

$$\boldsymbol{s}_L = \mathcal{G}\left(\boldsymbol{s}_0; \boldsymbol{\Omega}\right) = g_{\phi_{L-1}} \circ g_{\phi_{L-2}} \circ \cdots \circ g_{\phi_0}(\boldsymbol{s}_0)$$

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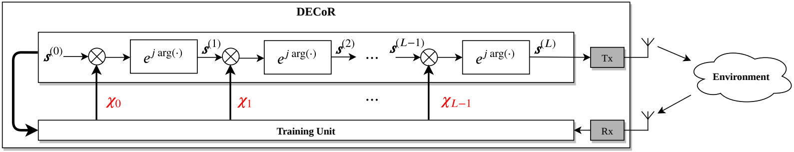

Figure: The proposed DECoR architecture for adaptive radar waveform design

DECoR Learning Algorithm

Step 0: (Initialization): Choose unimodular transmit sequence $\boldsymbol{s}_0 \in \mathbb{C}^N$, set $t = 0$, $\sigma = c$, $\delta \in (0, 1]$, $\boldsymbol{\Omega}^{(0)} = \{\boldsymbol{\chi}_i^{(0)}\}_{i=0}^{L-1}$ such that $\boldsymbol{\chi}_i^{(0)} \succ 0$, for $i \in \{0, \ldots, L-1\}$.

Step 1: (Random walk- generation): For $l \in \{0, \ldots, L-1\}$ and $i \in \{0, \ldots, B-1\}$ generate $\boldsymbol{D}_l^i \in \mathbb{C}^{N \times N}$ as the set of Hermitian positive-definite search direction matrices.

Step 2: (Random walk- perturbation): For $i \in \{0, \ldots, B-1\}$, form the parameter space $\boldsymbol{\Omega}^{(t)}$ as $\boldsymbol{\Omega}_i^{(t)} = \{\boldsymbol{\chi}_0^{(t)} + \boldsymbol{D}_0^i, \ldots, \boldsymbol{\chi}_{L-1}^{(t)} + \boldsymbol{D}_{L-1}^i\}$. Compute $\boldsymbol{s}_{L,i}^{(t)} = \mathcal{G}(\boldsymbol{s}_0; \boldsymbol{\Omega}_i^{(t)})$ for $i \in \{0, \ldots, B\}$ and $\boldsymbol{S}^{(t)} = \{\boldsymbol{s}_{L,0}^{(t)}, \ldots, \boldsymbol{s}_{L,B-1}^{(t)}\}$.

Step 3: (Collecting information): Transmit $\boldsymbol{S}^{(t)}$ and obtain $\boldsymbol{Y} = \{\boldsymbol{y}_0^{(t)}, \ldots, \boldsymbol{y}_{B-1}^{(t)}\}$. Compute $f(\boldsymbol{s})$ for each transmit/receive pair $(\boldsymbol{s}_{L,i}^{(t)}, \boldsymbol{y}_i^{(t)})$ and construct $\mathcal{F} = \{f(\boldsymbol{s}_{L,i}^{(t)})\}_{i=0}^{B-1}$.

Step 4: (Optimizing the DECoR architecture): Choose $i_\star$

$$i_\star = \underset{i \in [B]}{\arg\max} \; f(\boldsymbol{s}_{L,i}^{(t)}).$$

Update the network parameters if $f(\boldsymbol{s}_{L,i_\star}^{(t)}) \geq f(\boldsymbol{s}_L^{(t-1)})$ and set $\sigma \leftarrow c$. Otherwise, only update the search radius as $\sigma \leftarrow \delta\sigma$. Continue the online learning by going to Step 1.

Numerical Examples

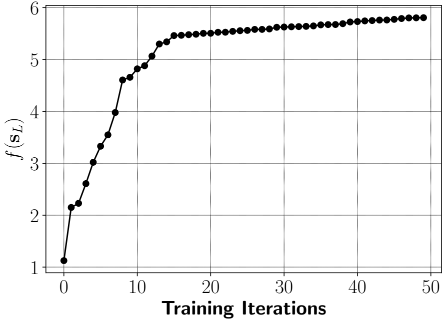

Figure: The objective value $f(s_L)$ of the DECoR vs. training iterations for a code length of $N = 10$

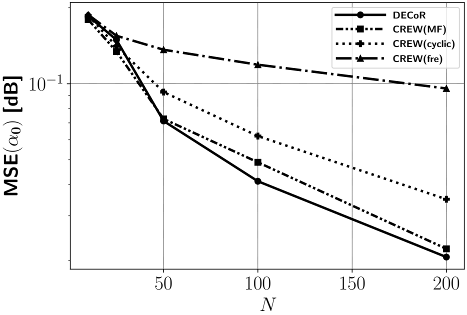

Figure: MSE values obtained by the different design algorithms for code lengths $N \in \{10, 25, 50, 100, 200\}$.

Summary

- In order to design cognitive radar waveforms, we bridge the gap between model-based and data driven techniques and propose a methodology to unfold the PMLI to solve a UQP
- Although the DECoR framework does not have access to the statistics of the environmental parameters, it is able to learn them by exploiting the observed data from interaction with the environment.

Thank you
and
Questions?